PACIFIC JOURNAL OF MATHEMATICS
Vol. 124, No. 2, 1986

ON INCLUSION RELATIONS FOR ABSOLUTE NÖRLUND SUMMABILITY

IKUKO MIYAMOTO

Recently Das gives sufficient conditions for $(N, r_n) \subseteq (N, p_n)(N, q_n)$ or $(N, p_n)(N, q_n) \subseteq (N, r_n)$, and for $|N, P_n| \sim |(N, p_n)(C, 1)|$. The purpose of this paper is to give sufficient conditions for $|N, r_n| \subseteq |(N, p_n)(N, q_n)|$ or $|(N, p_n)(N, q_n)| \subseteq |N, r_n|$. The results obtained here are also absolute summability analogues of Das' theorems.

1. Let $\{p_n\}$ and $\{q_n\}$ be real or complex sequences such that $P_n = \sum_{k=0}^n p_k \neq 0$ and $Q_n = \sum_{k=0}^n q_k \neq 0$. A sequence $\{s_n\}$ is said to be summable (N, p_n) to s, if $t_n^p = \sum_{k=0}^n p_{n-k} s_k / P_n \to s (n \to \infty)$, and summable $(N, p_n)(N, q_n)$ to s, if $t_n^{p,q} = \sum_{k=0}^n p_{n-k} t_k^q / P_n \to s (n \to \infty)$. It is said to be absolutely summable (N, p_n), or summable $|N, p_n|$, if $\sum |t_n^p - t_{n+1}^p| < \infty$.

Given two summability methods A and B, we write $A \subseteq B$ if each sequence summable A is summable B. If each includes the other, we write $A \sim B$.

We define the sequence $\{r_n\}$ by $r_n = \sum_{k=0}^n p_{n-k} q_k$ and define the sequence $\{c_n\}$ formally by $1/\sum_{n=0}^\infty p_n x^n = \sum_{n=0}^\infty c_n x^n$. We write $\{p_n\} \in \mathfrak{M}$ if $p_n > 0$, $p_{n+1}/p_n \leq p_{n+2}/p_{n+1} \leq 1$, and also write, for any sequence $\{f_n\}$, $f_n^{(1)} = \sum_{k=0}^n f_k$, $f_n^{(2)} = \sum_{k=0}^n f_k^{(1)}$. And K denotes an absolute constant, not necessarily the same at each occurrence.

On inclusion relations between two summability methods Das gives the following theorems.

THEOREM A [**1**, *Theorem* 2]. *If* $\{p_n\} \in \mathfrak{M}$ *and* $\{q_n\}$ *is positive, then* $(N, r_n) \subseteq (N, p_n)(N, q_n)$.

THEOREM B [**1**, *Theorem* 5]. *If* $\{p_n\} \in \mathfrak{M}$ *and* $\{q_n\}$ *is positive and* $(n+1) q_n = O(Q_n)$, *then* $(N, p_n)(N, q_n) \subseteq (N, r_n)$.

THEOREM C [**2**, *Theorem* 5]. *If* $\{p_n\} \in \mathfrak{M}$, *then* $|N, P_n| \sim |(N, p_n)(C, 1)|$.

The purpose of this paper is to prove the following theorems.