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APPROXIMATE GREEN FUNCTIONS AS A TOOL TO PROVE CORRECTNESS OF A FORMAL APPROXIMATION IN A MODEL OF COMPETING AND DIFFUSING SPECIES

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The purpose of this paper is two-fold; firstly, we shall derive some new results concerning a singular perturbation problem describing the stationary distribution of two competing and diffusing species; secondly, we want to demonstrate the power of a technique using approximate Green functions to prove the validity of a constructed formal approximation in a singular perturbation problem.

A mathematical model for the spatial distribution of two species consists of two coupled 2nd order O.D.E. on the interval $[-1,1]$ with Neumann boundary conditions:

$$\varepsilon^2 u'' = f(x,u,v), \qquad v'' = g(x,u,v) \tag{1.1}$$

$$u'(-1) = v'(-1) = 0, \quad u'(1) = v'(1) = 0. \tag{1.2}$$

Here, ε is a small parameter > 0 and $'$ denotes derivation w.r.t. the x-variable.

This model (or rather its time-dependent version) has been proposed by several authors to explain the coexistence of competing and diffusing species in some subdomain and not elsewhere, cf. [**4**], [**9**], [**5**], [**6**], [**10**]. For certain non-linearities f, g there is the possibility of a solution with a sharp transition phenomenon in the u-variable at an internal point $y \in (-1,1)$. The domain $(-1,1)$ is then subdivided in two subdomains $(-1, y)$ and $(y, 1)$ where u behaves essentially different. At y the jump in the u-behaviour is smoothed by an internal layer.

These results are derived by constructing an asymptotic approximation for $\varepsilon \downarrow 0$ of the solution of (1.1–2). In [**4**], only a first approximation without detailed information on the internal layer is used. In [**9**], all over the domain higher order terms are included in the construction. This is done by dealing with two separate problems on $[-1, y + \delta(\varepsilon)]$ and $[y + \delta(\varepsilon), 1]$ with Dirichlet boundary conditions at the transition point. Free constants introduced in this way, such as $\delta(\varepsilon)$, are determined later on by requiring a smooth connection at the transition point. In this work a restriction has to be made about the non-linearity g (see [**9**], assumption