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ON PELCZYNSKI'S PROPERTIES (V) AND (V*)

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It is shown that a Banach lattice X has Pelczynski's property (V*) if and only if X contains no subspace isomorphic to c_0. This result is used to show that there is a Banach space E that has Pelczynski's property (V*) but such that its dual E^* fails Pelczynski's property (V), thus answering in the negative a question of Pelczynski.

In his fundamental paper [7], Pelczynski introduced two properties of Banach spaces, namely property (V) and property (V*). For a Banach space X we say that *X has property* (V*) if any subset $K \subset X$ such that $\lim_n \sup_{x \in K} x_n^*(x) = 0$ for every weakly unconditionally Cauchy series (w.u.c.) $\sum_{n=1}^{\infty} x_n^*$ in X^*, then K is relatively weakly compact. We say that *X has property* (V) if any subset $K \subset X^*$ such that $\lim_n \sup_{x^* \in K} x_n(x^*) = 0$ for every weakly unconditionally Cauchy series (w.u.c.) $\sum_{n=1}^{\infty} x_n$ in X then K is relatively weakly compact. In [7] Pelczynski noted that it follows directly from the definition that if X^* has property (V) then X has property (V*), and he asked [7, Remark 3, p. 646] if the converse is true. As we shall soon show Example 5 below will provide a negative answer to Pelczynski's question.

In this paper we will concentrate on property (V*) and we shall refer the reader to [4] and [7] for more on property (V). Among classical Banach spaces that have property (V*), L^1-spaces are the most notable ones. In [7] Pelczynski showed that if a Banach space has property (V*), then it must be weakly sequentially complete. He also-noted that for a closed subspace X of a space with unconditional basis, the space X has property (V*) if and only if X contains no subspace isomorphic to c_0. This prompted the following natural question:

Problem 1. *Let $(\Omega, \Sigma, \lambda)$ be a probability space, and let X be a closed subspace of a Banach space with unconditional basis. Does the Banach space $L^1(\lambda, X)$ of Bochner integrable X-valued functions have property* (V*) *whenever X has* (V*)?

In this paper we shall give an affirmative answer to this question, in fact we shall prove a more general result, namely if X is a separable subspace of an order continuous Banach lattice, then $L^1(\lambda, X)$ has