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COUNTING FUNCTIONS AND MAJORIZATION FOR JENSEN MEASURES

CHARLES S. STANTON

We establish a generalization for uniform algebras of the classical identities of Hardy and Stein. We use this and an estimate based on the isoperimetric inequality to give a proof of H. Alexander's spectral area theorem. We use similar methods to prove a theorem of Axler and Shapiro about VMOA of the unit ball in $\mathbf{C}^n$.

1. Introduction. Given a Jensen measure on the maximal ideal space of A, we introduce a "counting function" analogous to the classical counting function $N(r, w)$ of Nevanlinna's value distribution theory. In particular, this counting function is non-negative, supported on the spectrum of f, and a subharmonic function of w on the complex plane except for a logarithmic pole. We next establish an identity for integral means of f in terms of this counting function. This generalizes Theorems 2 and 9 of **[6]**. Classical identities of Cartan and of Hardy and Stein occur as special cases.

As an application, we give a proof (for Jensen measures) of H. Alexander's spectral area estimate:

THEOREM A **[1, 2]**. *Let A be a uniform algebra, $\varphi \in M_A$, and σ a Jensen measure for φ. Then*

$$\int_{M_A} |f|^2 \, d\sigma \leq \frac{1}{\pi} \operatorname{area}(\operatorname{spec} f) + |f(\varphi)|^2. \tag{1}$$

Finally, we apply these counting function techniques to prove a slight generalization of the following result of Axler and Shapiro about analytic functions of vanishing mean oscillation (VMOA) of the unit ball in $\mathbf{C}^n$.

THEOREM B [3]. *Suppose $f \in H^\infty(\mathbf{B}^n)$ and for each $\zeta \in S$*

$$\operatorname{area}(\operatorname{cl}(f, \zeta)) = 0.$$

Then $f \in$ VMOA.

2. Uniform algebras and Jensen measures. We first recall some basic facts about uniform algebras and Jensen measures (for more details