PACIFIC JOURNAL OF MATHEMATICS
Vol. 125, No. 2, 1986

HIGHLY TRANSITIVE GROUP ACTIONS ON TREES AND NORMALIZING TITS SYSTEMS

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The theory of Tits systems associates to each group G with Tits system a simplicial complex, together with a 'numbering' of its set of vertices, on which the group acts in a highly transitive manner. This numbered simplicial complex is a tree if and only if the Weyl group of the Tits system is an infinite dihedral group, for example when G is $\mathrm{PSL}(2, K)$, K is a local field, with its affine Tits system structure, or when G is the central quotient of the group associated to a Kac-Moody Lie algebra of rank 1. There are non-algebraic examples of such groups as well, such as the full automorphism group of a numbered tree.

In this paper, we investigate the structure of groups acting highly transitively on a tree without preserving a given numbering of the set of vertices. Such groups no longer possess the structure of a Tits system. However, we show that such groups have a pair of subgroups B and N which satisfy all the properties of a Tits system, except the requirement that the generators of the Weyl group should not normalize B. We have called a group G with B and N satisfying these properties a normalizing Tits system. We show that these groups have some properties closely related to, but different from, those arising in the theory of Tits systems, such as the structure of the set of 'parabolic subgroups' of G. There are very simple examples of such groups, for instance the full group of automorphisms of a tree of $\mathrm{Gl}(2, K)$, K a local field.

The most important property familiar from the theory of Tits systems which still holds for the groups we study here is the existence of a Bruhat decomposition. However, while the Weyl group is still a Coxeter group, with a distinguished set S of generators, the rule of multiplying double cosets by elements of S is very different from the familiar situation: there are elements s in S for which for all w in the Weyl group, $s.B.w.B = B.s.w.B$.

For the theory of Tits systems and some of its applications, the reader can consult Tits, Bruhat and Tits, Iwahori and Matsumoto and Garland and the bibliographies referred to therein. The idea of studying groups with Bruhat decomposition more general than those with Tits system was first introduced in our work. It was in that paper that the intimate relation between multiple transitivity and groups with Bruhat decomposition was first noticed. But while this paper is thus conceptually closely related to our work, the notion of transitivity here introduced and the proof of Bruhat decomposition are wholly different. Also, the central notion of this paper—that of a normalizing Tits system—is new.

0. Introduction. Trees arise in a variety of mathematical contexts (see [Serre]). They are the simplest examples of affine buildings (for the general theory of buildings see [T], for affine buildings and their origin,