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NECESSARY AND SUFFICIENT CONDITIONS FOR SIMPLE A-BASES

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Let A be a set of m distinct integers with $m \geq 2$ and $0 \in A$. It is shown that A possesses a simple A-base if and only if A is a complete residue system modulo m and the elements of A are relatively prime.

The notions of simple and non-simple A-bases, due to de Bruijn, are defined as follows.

DEFINITION 1. Let A be as above. The integral sequence $B = \{b_i\}_{i \geq 1}$ is called an A-base for the set of integers provided that every integer n can be represented uniquely in the form

$$n = \sum_{i=1}^{r(n)} a_i b_i, \qquad a_i \in A \ \forall i.$$

If (with possible rearrangement) B can be written in the form $B = \{d_i m^{i-1}\}_{i \geq 1}$ where the d_i are integers, then it is called a simple A-base.

The notion of an A-base was generalized by Long and Woo to that of an $\mathfrak{A}$-base where $\mathfrak{A} = \{A_i\}$ and each A_i is a set of m_i distinct integers with $0 \in A_i$ and $m_i \geq 2$ for all i. The definition is as follows.

DEFINITION 2. Let $\mathfrak{A}$ be as above. The integral sequence $B = \{b_i\}_{i \geq 1}$ is called an $\mathfrak{A}$-base for the set of integers provided every integer n can be written uniquely in the form

$$n = \sum_{i=1}^{r(n)} a_i b_i, \qquad a_i \in A_i \ \forall i.$$

If (with possible rearrangement) B can be written in the form $B = \{d_i M_{i-1}\}_{i \geq 1}$ where the d_i are integers and where $M_0 = 1$ and $M_i = \prod_{j=1}^{i} m_j$ for $i \geq 1$, then it is called a simple $\mathfrak{A}$-base.

De Bruijn has pointed out that it is not yet known for which A's there exist simple A-bases nor it is known for which A's there exist non-simple A-bases. He gives several examples and then observes that if A has a simple A-base it is necessary that A form a complete residue system