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REALIZING CERTAIN POLYNOMIAL ALGEBRAS AS COHOMOLOGY RINGS OF SPACES OF FINITE TYPE FIBERED OVER $\times BU(d)$

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The problem of constructing topological spaces whose cohomology ring with coefficients in the field of p elements is a polynomial algebra has attracted the attention of algebraic topologists for many decades. Apart from the naturally occurring examples, classifying spaces of Lie groups away from their torsion primes, rather little progress was made until the construction of Clark and Ewing of a vast number of new non-modular examples. The completeness of their construction in the non-modular case was shown by Adams and Wilkerson (see Smith and Switzer for a compact-proof). One interest in the construction of spaces with polynomial cohomology is that they are related to the study of finite H-spaces, which should appear as their loop spaces; "should" because the construction of Clark and Ewing does not yield a simply connected CW complex of finite type. On the contrary the construction of Clark and Ewing yields non-simply connected spaces that are p-adically complete. By forming their finite completion they can be made simply connected. But considerably more effort would be required to show that they have the homotopy type of the p-completion of a simply connected CW complex of finite type.

We will avoid these drawbacks by constructing for certain of the examples of Clark and Ewing a simply connected space of finite type with the requisite cohomology.

Recall that the construction of [**6**] depends on a group $G < \mathrm{GL}(V)$, $V = \oplus_n \mathbf{F}_p$, where $\mathbf{F}_p$ is the field of p-elements, and which satisfies: G is generated by pseudo reflections and $|G| \not\equiv O(p)$. A theorem of Chevalley [**2**; V §5 no. 5, 3, Thm. 3] [**24**] shows that:

$$P(V^*)^G \simeq P[\rho_1, \ldots, \rho_n]$$

where $P(V^*)$ denotes the ring of polynomials on the dual vector space V^* of V upon which G acts, and $P(V^*)^G \leq P(V^*)$ is the ring of invariant polynomials. We will say that $P(V^*)^G$ satisfies the *weak splitting principle* iff we can find polynomial generators $\rho_1, \ldots, \rho_n \in P(V^*)^G$, and polynomials $f_1(X), \ldots, f_b(X) \in P(V^*)^G[X]$, where X is an indeterminate of degree 2, such that:

(1) $\rho_1, \ldots, \rho_n$ are among the coefficients of $f_1(X), \ldots, f_b(X)$, and