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BIJECTIVE PROOFS OF BASIC HYPERGEOMETRIC SERIES IDENTITIES

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Bijections are given which prove the following theorems: the q-binomial theorem, Heine's ${}_2\Phi_1$ transformation, the q-analogues of Gauss', Kummer's, and Saalschütz's theorems, the very well poised ${}_4\Phi_3$ and ${}_6\Phi_5$ evaluations, and Watson's transformation of an ${}_8\Phi_7$ to a ${}_4\Phi_3$. The proofs hold for all values of the parameters. Bijective proofs of the terminating cases follow from the general case. A bijective version of limiting cases of these series is also given. The technique is to mimic the classical proofs, based upon a bijective proof of the q-binomial theorem and sign-reversing involutions which cancel infinite products.

1. Introduction. In 1969 George Andrews [**1**] began to develop a calculus for partition functions. His stated goal was to "...translate a sizable portion of the techniques of the elementary theory of basic hypergeometric series into arithmetic terms". Ideally, he wanted to prove any theorem in basic hypergeometric series by a bijection. In this paper we show (under certain requirements) that this can be accomplished.

Andrews' main object was to give bijective proofs of partition theorems (such as the Rogers-Ramanujan identities). It was well-known that these theorems were closely related to basic hypergeometric series. If a bijective proof of a partition theorem were desired, could one possibly give a bijective proof of a related basic hypergeometric series? If each step in the manipulation of a basic hypergeometric series could be interpreted bijectively, the result would be a bijective proof of the partition theorem.

Andrews gave a bijective proof of the q-binomial theorem, which is the cornerstone of basic hypergeometric series. He also showed how to combinatorially interpret cancellations of infinite products, a manipulation of basic hypergeometric series which occurs frequently. However, he used the principle of inclusion-exclusion which, strictly speaking, is not bijective because it cancels objects in clumps. From the recent work of Gessel-Viennot [**13**] and Garsia-Milne [**11**] the appropriate bijective replacement for the principle of inclusion-exclusion is a sign-reversing involution. So this part of the theory of basic hypergeometric series can now be done bijectively, which is the main purpose of this paper.