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ON THE UNIVERSALITY OF SYSTEMS OF WORDS IN PERMUTATION GROUPS

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In the classes of infinite symmetric groups, their normal subgroups, and their factor groups, we determine those groups which are equivalent in the sense that they may not be distinguished by the solvability of a system of finitely many equations in variables and parameters.

1. Introduction and results. Recently, several authors [**1,3–5,8, 9,12**] studied the solvability of equations of the form $w(x_1,\ldots,x_n) = y$, where w is a group word, in various kinds of groups. In [**1,3,12**] this problem was considered for infinite symmetric groups. Here we consider the simultaneous solvability of several equations of a similar form in infinite symmetric groups, their normal subgroups, and their factor-groups.

Let G be a group, $x_1,\ldots,x_n$ variables, $y_1,\ldots,y_m$ parameters, and $w_i = w_i(x_1,\ldots,x_n; y_1,\ldots,y_m)$ $(i \in I)$ group words in these variables and parameters. We say that $W = \{w_i \mid i \in I\}$ is G-universal if G satisfies the following property:

For all $y_1,\ldots,y_m \in G$ there exist $x_1,\ldots,x_n \in G$ such that for all $i \in I$, $w_i(x_1,\ldots,x_n; y_1,\ldots,y_m) = e$.

Two groups G and H will be called equationally equivalent, $G \equiv_{\mathrm{eq}} H$, if for any finite set W of words w_i as above, W is G-universal iff W is H-universal.

Let S_ν denote the infinite symmetric group of all permutations of a set of cardinality $\aleph_\nu$ and, for $0 \le \tau \le \nu + 1$, S_ν^τ its normal subgroup comprising all permutations moving less than $\aleph_\tau$ elements of the underlying set. The problem of the elementary equivalence (definability) of the groups S_ν $(\nu \ge 0)$ was solved in Shelah [**11**]. Here we will consider the problem of the equational equivalence of the groups S_ν. A very similar problem was suggested by J. Isbell, cf. [**6**; p. 20]. Throughout this paper, let $V = \{v_1, v_2, v_3\}$ be the following set of words in parameters y_1, y_2 and variables x_1, x_2, x_3:

$$v_i = y_i^{-1} \cdot x_1^{-1} \cdot y_i \cdot x_1 \quad (i = 1,2),$$

$$v_3 = y_1^{-1} \cdot x_2^{-1} \cdot x_1 \cdot x_2 \cdot x_3^{-1} \cdot x_1 \cdot x_3.$$