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OSCILLATORY PROPERTIES OF SYSTEMS OF FIRST ORDER LINEAR DELAY DIFFERENTIAL INEQUALITIES

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Sufficient conditions are obtained for the nonexistence of eventually positive bounded solutions of the system of delay differential inequalities

$$\frac{dx_i(t)}{dt} + \sum_{j=1}^{n} a_{ij}x_j(t-\tau_{ij}) \leq 0; \qquad i = 1,2,\ldots,n$$

and for the nonexistence of eventually negative bounded solutions of

$$\frac{dx_i(t)}{dt} + \sum_{j=1}^{n} a_{ij}x_j(t-\tau_{ij}) \geq 0; \qquad i = 1,2,\ldots,n.$$

As a corollary to the above we obtain sufficient conditions for all bounded solutions of

$$\frac{dx_i(t)}{dt} + \sum_{j=1}^{n} a_{ij}x_j(t-\tau_{ij}) = 0; \qquad i = 1,2,\ldots,n$$

to be oscillatory.

1. Introduction. The oscillatory and asymptotic behaviour of scalar delay differential equations and inequalities has been the subject of numerous investigations. For a recent survey of results we refer to Zhang [**20**]. First order differential inequalities with delayed arguments have been discussed by Ladas and Stavroulakis [**9**] and Stavroulakis [**18**]. The purpose of this brief article is to derive a set of sufficient conditions for all bounded solutions of a linear system of the type

$$(1.1) \qquad \frac{dx_i(t)}{dt} + \sum_{j=1}^{n} a_{ij}x_j(t-\tau_{ij}) = 0 \qquad i = 1,2,\ldots,n;\ t > t_0$$

to be "oscillatory" by considering the twin systems of inequalities

$$(1.2) \qquad \frac{dx_i(t)}{dt} + \sum_{j=1}^{n} a_{ij}x_j(t-\tau_{ij}) \leq 0 \qquad i = 1,2,\ldots,n;\ t > t_0$$

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