PACIFIC JOURNAL OF MATHEMATICS
Vol. 129, No. 1, 1987

THE HARMONIC REPRESENTATION OF $U(p,q)$ AND ITS CONNECTION WITH THE GENERALIZED UNIT DISK

MARK G. DAVIDSON

In this paper we study the very close connection between the kth tensor product of the harmonic representation ω of $U(p,q)$ and the generalized unit disk $\mathscr{D}$. We give a global version of ω realized on the Fock space as an integral operator. Each irreducible component of ω is shown to be equivalent in a natural way to a multiplier representation of $U(p,q)$ acting on a Hilbert space $\mathscr{H}(\mathscr{D},\lambda)$ of vector-valued holomorphic functions on $\mathscr{D}$. The intertwining operator between these realizations is then explicitly constructed. We determine necessary and sufficient conditions for square integrability of each component of ω and in this case derive the Hilbert space structure on $\mathscr{H}(\mathscr{D},\lambda)$.

Introduction. Of interest here are the diverse roles the generalized unit disk plays in the constructions mentioned above. Our principal objective is to give a disk picture realization of all $U(p,q)$ highest weight modules. This is done in §3. Further, we are interested in their unitary structure. We will say more on that later.

In the literature various versions of $U(p,q)$ highest weight modules appear. Typical are constructions involving the Siegal upper half plane [**4**, **8**] or the open set of positive p-planes in the Grassmannian [**12**]. More recently, Patton and Rossi [**13**] have used cohomological methods to realize these modules and the Penrose transform has related these to other constructions (cf. also [**12, 14**]). Most notable, however, is the paper of Kashiwara and Vergne [**8**]. There they decompose ω (we will use ω to mean the kth tensor product of the standard Segal-Shale-Weil representation of $U(p,q)$) and produce, as they conjectured, all highest weight $U(p,q)$ modules on a Schroedinger-Fock space (cf. [**2, 7**]). In their version ω is constructed by determining its action on certain subgroups whose product is dense in $U(p,q)$. Together these actions lead to a unitary representation of the whole group. Their main results are the decomposition of ω into its irreducible components ω_λ, $\lambda \in \Lambda \subseteq U(k)\hat{}$, and an explicit description of Λ in terms of the signature of irreducible representations of the dual group $U(k)$.