

## THE EULER CHARACTER AND CANCELLATION THEOREMS FOR WEYL MODULES

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We consider for a simple, simply connected algebraic group over an algebraically closed field of characteristic  $p$ , modules induced from characters on a Borel subgroup. We ask if the socle levels of the modules induced from characters in a general position determine the socle levels of modules induced from characters in a singular position. Technically, the question may be phrased in terms of the infinitesimal group subscheme determined by the Frobenius morphism of the global group. Qualitatively, we show that the socle levels of the global induced modules are induced from the socle levels of the infinitesimal induced modules, assuming only that Bott's Theorem applies. Quantitatively, we show that the multiplicities of the composition factors of the module induced from an infinitesimal socle layer are determined by the structure of the layer as a module for the Borel subgroup.

**Introduction.** Let  $G$  be a simple, simply connected algebraic group over an algebraically closed field of characteristic  $p$ , and let  $T$  be a maximal torus in a Borel subgroup  $B$ . Let  $G_n$  be the kernel of the  $n$ th power of the Frobenius map on  $G$ . Let  $H^0(G/B, \lambda)$  be the  $G$ -module induced from a character  $\lambda$  on  $B$ ,  $H^0(G_n B/B, \lambda)$  the  $G_n B$ -module induced from  $\lambda$ , and  $H^0(G/G_n B, M)$  the  $G$ -module induced from a  $G_n B$ -module  $M$ . We consider whether the socle levels of  $H^0(G/B, \lambda)$  for  $\lambda$  in a singular position can be computed in terms of the socle levels of  $H^0(G/B, \mu)$  for  $\mu$  in general position. Equivalently, one can ask if the socle levels of  $H^0(G/B, \lambda)$  can be computed in terms of the socle levels of  $H^0(G_n B/B, \lambda)$ . Qualitatively, we show that the socle levels of  $H^0(G/B, \lambda)$  are induced from the  $G_n$ -socle levels of  $H^0(G_n B/B, \lambda)$ , assuming only that Bott's Theorem applies to the composition factors of  $H^0(G_n B/B, \lambda)$ . That leaves open the quantitative question of relative multiplicities of the composition factors of the socle levels of  $H^0(G/B, \lambda)$  and  $H^0(G_n B/B, \lambda)$ . We do show that the multiplicities of the composition factors of the  $G$ -module induced from a  $G_n$ -socle layer  $M$  of  $H^0(G_n B/B, \lambda)$  are determined by the  $G_n B$ -structure of the  $G_n$ -semisimple module  $M$ .