PACIFIC JOURNAL OF MATHEMATICS
Vol. 135, No. 1, 1988

INTEGRATED SEMIGROUPS AND THEIR APPLICATIONS TO THE ABSTRACT CAUCHY PROBLEM

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This paper is concerned with characterizations of those linear, closed, but not necessarily densely defined operators A on a Banach space E with nonempty resolvent set for which the abstract Cauchy problem $u'(t) = Au(t)$, $u(0) = x$ has unique, exponentially bounded solutions for every initial value $x \in D(A^n)$.

Investigating these operators we are led to the class of "integrated semigroups". Among others, this class contains the classes of strongly continuous semigroups and cosine families and the class of exponentially bounded distribution semigroups.

The given characterizations of the generators of these integrated semigroups unify and generalize the classical characterizations of generators of strongly continuous semigroups, cosine families or exponentially bounded distribution semigroups.

We indicate how integrated semigroups can be used studying second order Cauchy problems $u''(t) - A_1u'(t) - A_2u(t) = 0$, operator valued equations $U'(t) = A_1U(t) + U(t)A_2$ and nonautonomous equations $u'(t) = A(t)u(t)$.

1. Introduction. We study integrated semigroups and their connection to the abstract Cauchy problem

$$\text{(ACP)} \qquad u'(t) = Au(t); \qquad u(0) = x$$

where A is a linear, closed operator on a Banach space E with nonempty resolvent set and domain $D(A)$.

A function $u(\cdot)\colon [0,\infty) \to D(A)$ with $u(\cdot) \in C^1([0,\infty), E)$ and $u(0) = x$ which satisfies (ACP) is called a solution of (ACP).

Studying (ACP), we will introduce the notion of "generators of integrated semigroups". If A is such a generator, then (ACP) is exponentially wellposed in the following sense: there exist an $n \in N$ and constants M, w, such that, for all $x \in D(A^n)$, there exists a unique solution $u(\cdot)$ of (ACP) with $|u(t)| \leq Me^{wt}|x|_{n-1}$ for all $t \geq 0$, where $|x|_{n-1} := |x| + |Ax| + \cdots + |A^{n-1}x|$ denotes the graph norm of the Banach space $[D(A^{n-1})]$ (for a more refined definition of wellposedness, see Def. 3.2).