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SYSTEMS OF NONLINEAR WAVE EQUATIONS WITH NONLINEAR VISCOSITY

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An equation of the form

$$\ddot{u} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial W(p)}{\partial p_i} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial V(q)}{\partial q_i} = f$$

where $p = \nabla u$, $q = \nabla \dot{u}$, $\dot{u} = \partial u/\partial t$, $\ddot{u} = \partial^2 u/\partial t^2$ represents, for suitable functions $W(p)$, $V(q)$, a nonlinear hyperbolic equation with nonlinear viscosity and it appears in models of nonlinear elasticity. In this paper existence and regularity of solutions for the Cauchy problem will be established. In particular, if $n = 2$, or if $n \geq 3$ and the eigenvalues of $(\partial^2 V/\partial q_j \partial q_j)$ belong to a "small" interval, then the solution is classical. These results will actually be established for a system of equations of the above type.

Introduction. Consider a system of N nonlinear equations

$$\ddot{u}_k - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial W(p)}{\partial p_{ki}} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial V(q)}{\partial q_{ki}} = f_k \qquad (1 \leq k \leq N) \tag{0.1}$$

in a cylinder $\Omega \times (0, \infty)$, with initial data

$$u_k(x, 0) = u_{k0}(x), \quad \dot{u}_k(x, 0) = u_{k1}(x) \tag{0.2}$$

and boundary conditions

$$u = 0 \quad \text{if } x \in \partial\Omega,\ t > 0; \tag{0.3}$$

here Ω is a bounded domain in $\mathbf{R}^n$,

$$p = (p_{li}), \quad q = (q_{li}) \quad \text{and}$$
$$p_{li} = \frac{\partial u_l}{\partial x_i}, \quad q_{li} = \frac{\partial \dot{u}_l}{\partial x_i}, \quad \dot{w} = \frac{\partial w}{\partial t}.$$

The special case

$$N = 1, \quad \sum_{i=1}^{n} \frac{\partial}{\partial x_i} \frac{\partial V(q)}{\partial q_{ki}} = \Delta \dot{u} \qquad (k = 1) \tag{0.4}$$

has been studied by several authors. For $n = 1$, existence and uniqueness of a classical solution was established in [1], [2], [6], [7]. For