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THE ISOMORPHISM PROBLEM FOR ORTHODOX SEMIGROUPS

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The author's structure theorem for orthodox semigroups produced an orthodox semigroup $\mathscr{H}(E, T, \psi)$ from a band E, an inverse semigroup T and a morphism ψ between two inverse semigroups, namely T and W_E/γ, an inverse semigroup constructed from E. Here, we solve the isomorphism problem: when are two such orthodox semigroups isomorphic? This leads to a way of producing all orthodox semigroups, up to isomorphism, with prescribed band E and maximum inverse semigroup morphic image T.

1. Preliminaries. A semigroup S is called *regular* (in the sense of von Neumann for rings) if for each $a \in S$ there exists $x \in S$ such that $axa = a$; and S is called an *inverse semigroup* if for each $a \in S$ there is a unique $x \in S$ such that $axa = a$ and $xax = x$. A *band* is a semigroup in which each element is idempotent, and an *orthodox semigroup* is a regular semigroup in which the idempotents form a subsemigroup (that is, a band).

We follow the notation and conventions of Howie [**4**].

Result 1 [**3**, *Theorem* 5]. *The maximum congruence contained in Green's relation $\mathscr{H}$ on any regular semigroup S, $\mu = \mu(S)$ say, is given by $\mu = \{(a, b) \in \mathscr{H}$: for some [for each pair of] $\mathscr{H}$-related inverses a' of a and b' of b, $a'ea = b'eb$ for each idempotent $e \leq aa'\}$.*

A regular semigroup S is called *fundamental* if μ is the identity relation on S. For each band E, the semigroup W_E is fundamental, orthodox, has its band isomorphic to E, and contains, for each orthodox semigroup S with band E, a copy of S/μ as a subsemigroup: see the author [**1**] (or [**3**] with $E = \langle E\rangle$ and $W_E = T_{\langle E\rangle}$) or Howie [**4**, §VI.2].

Now take any inverse semigroup T, and, if such exist, any idempotent-separating morphism $\psi: T \to W_E/\gamma$ whose range contains the semilattice of all idempotents of W_E/γ, where γ denotes the least inverse semigroup congruence on W_E. A semigroup $\mathscr{H}(E, T, \psi)$ (see