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THE C^*-ALGEBRAS ASSOCIATED WITH MINIMAL HOMEOMORPHISMS OF THE CANTOR SET

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We investigate the structure of the C^*-algebras associated with minimal homeomorphisms of the Cantor set via the crossed product construction. These C^*-algebras exhibit many of the same properties as approximately finite dimensional (or AF) C^*-algebras. Specifically, each non-empty closed subset of the Cantor set is shown to give rise, in a natural way, to an AF-subalgebra of the crossed product and we analyze these subalgebras. Results of Versik show that the crossed product may be embedded into an AF-algebra. We show that this embedding induces an order isomorphism at the level of K_0-groups. We examine examples arising from the theory of interval exchange transformations.

1. Preliminaries. We begin with an introduction to some terminology and notation, and a description of the results.

Throughout, we will let X denote the Cantor set. That is, X is a totally disconnected compact metrizable space with no isolated points. Generally, for any compact Hausdorff space, Z, we let $C(Z)$ denote the C^*-algebra of continuous complex-valued functions on Z.

We say a subset E of X is clopen if it is both open and closed. We let χ_E denote the characteristic function of E, which will be continuous if E is clopen. A partition, $\mathscr{P}$, of X we define to be a finite collection of pairwise disjoint clopen sets whose union is all of X. If $\mathscr{P}$ is a partition of X, we let $\mathscr{C}(\mathscr{P}) = \operatorname{span}\{\chi_E | E \in \mathscr{P}\}$. $\mathscr{C}(\mathscr{P})$ may be viewed as those functions in $C(X)$ which are constant on each element of $\mathscr{P}$. The fact that X is totally disconnected implies that any function in $C(X)$ may be approximated by one in some $\mathscr{C}(\mathscr{P})$. Given two partitions $\mathscr{P}_1$ and $\mathscr{P}_2$, of X, we say $\mathscr{P}_2$ is finer than $\mathscr{P}_1$ and write $\mathscr{P}_2 \geq \mathscr{P}_1$, if each element of $\mathscr{P}_2$ is contained in a single element of $\mathscr{P}_1$. This is clearly equivalent to the condition that $\mathscr{C}(\mathscr{P}_1) \subset \mathscr{C}(\mathscr{P}_2)$. Given two partitions $\mathscr{P}_1$ and $\mathscr{P}_2$, we define the partition $\mathscr{P}_1 \vee \mathscr{P}_2$ to be $\{E \cap F | E \in \mathscr{P}_1, F \in \mathscr{P}_2\}$.

We let φ be a homeomorphism of X which we shall always assume to be minimal. That is, there are no closed φ-invariant sets except for the empty set and X itself. This is equivalent to the condition that, for any point x in X, the set $\{\varphi^n(x) | n \geq 0\}$ is dense in X. We shall refer to