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ON KNOT INVARIANTS RELATED TO SOME STATISTICAL MECHANICAL MODELS

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Dedicated to the memory of Henry Dye

We use three different kinds of statistical mechanical models to construct link invariants. The vertex models emerge as the most general. Our treatment of them is essentially the same as Turaev's. Using the work of Goldschmidt we are able to define models whose invariants are homology invariants for branched covers. Thus the statistical mechanical framework embraces both the "classical" and the "new" link invariants.

0. Introduction. In this paper we shall discuss three types of statistical mechanical models—vertex models, Potts type models, and IRF models. In all cases we shall see that the models may be defined on a knot diagram (replacing the lattice of the model), and that a suitable variation on the partition function of the system is often a knot invariant, i.e. depends only on the knot as a three-dimensional entity and not on the chosen diagram.

The connection between knot theory and statistical mechanics was first established, indirectly, in **[J1]** where it was observed that the Temperley-Lieb algebra of the Potts and ice-type models (see **[Ba]** and **[TL]**) can be used to define a knot invariant using the theory of braids and a certain trace on the Temperley-Lieb algebra, discovered in the course of investigations into type II_1 factors (see **[J2]**). (This invariant is a Laurent polynomial in $\sqrt{t}$ which we shall write $V_L(t)$, where L is some oriented link.) But it was Kauffman who first began to understand this connection in a direct way with his "states model" for V_L, which freed the understanding of V_L from the use of braids or inductive methods (as in [F+]). Kauffman's model seems very special to V_L, but another approach to such explicit formulae was suggested by the braid formalism. The author succeeded in "unbraiding" a trace formula for a series of specializations of the two variable polynomial of [F+]. The relevant braid group representations were discovered by Jimbo [Ji], Drinfeld **[D]**, and Wenzl **[W1]**. This unbraiding was reported in a letter to Kauffman and we give the details of it in this paper. It was immediately generalized by Turaev **[Tu]** to embrace the Kauffman polynomial. We present our own version of Turaev's