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OPERATORS WHICH SATISFY POLYNOMIAL GROWTH CONDITIONS

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Consider the class of bounded linear operators S such that $\|\exp(itS)\|$ has polynomial growth in $|t|$ on $\mathbf{R}$. In this paper it is shown that the operators in this class have many interesting properties in common with selfadjoint operators.

1. Introduction. If S is a bounded linear selfadjoint operator on Hilbert space, then $\exp(itS)$ is a unitary operator for all $t \in \mathbf{R}$, and thus

$$\|\exp(itS)\| = 1 \qquad (t \in \mathbf{R}). \tag{1}$$

When S is an operator on a Banach space for which (1) holds, then S is called Hermitian. The class of Hermitian operators has proved useful in the study of spectral operators. In this paper we study a more general class of operators, those for which the growth of $\|\exp(itS)\|$ is at most polynomial in $t \in \mathbf{R}$, explicitly:

$$\exists K > 0 \quad \text{and} \quad \exists \delta \geq 0 \quad \text{such that } \|\exp(itS)\| \leq K(1 + |t|^{\delta}) \qquad (t \in \mathbf{R}). \tag{2}$$

Although this is a special class of operators, it does contain many interesting examples, and useful properties can be proved for operators in this class.

Throughout this paper X is a Banach space. All operators on X are automatically assumed to be linear and bounded. Let $\mathscr{P}(X)$ denote the set of all operators on X for which (2) holds. Here is a list of some types of operators in $\mathscr{P}(X)$:

A. Hermitian or Hermitian equivalent operators.
B. Operators on a Hilbert space of the form TRS where $R \geq 0$ and ST is selfadjoint.
C. Well-bounded operators (T is well-bounded means that for some interval $[a, b]$, $\exists K > 0$, such that for all polynomials p, $\|p(T)\| \leq K(|p(b)| + \int_a^b |p'(t)|\, dt)$).
D. Nilpotent and projection operators.