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# EQUIVARIANT ORIENTATIONS AND $G$-BORDISM THEORY

STEVEN R. COSTENOBLE AND STEFAN WANER

**We outline a new geometric theory of orientations under the action of a group $G$ and formulate bordism theories for $G$-oriented manifolds. These theories extend the classical $G$-bordism theories (graded on $\mathbb{Z}$), as well as the $\mathrm{RO}(G)$-graded oriented $G$-bordism theories which describe bordism of $G$-manifolds with restricted local representation structure. The theories we obtain account for oriented and unoriented bordism of $G$-manifolds with and without restricted local representation structure. We further obtain spectral sequences converging to these theories through adjacent family constructions.**

**1. Introduction and statement of results.** The literature contains divergent views both on the notion of equivariant dimension and on equivariant orientability for smooth actions of transformation groups.

As to the former, one has firstly the classical $G$-bordism groups **[D2, S2]** graded on $\mathbb{Z}$, where one regards the dimension of a smooth $G$-manifold as an integer in the usual sense. Secondly, one has the $\mathrm{RO}(G)$-graded $G$-bordism theories of Pulikowski, Kosniowski and the authors **[P2, K1, C1, W1]**, where one considers $G$-manifolds modelled on a fixed virtual representation of $G$, and regards the dimension of a $G$-manifold as the element of $\mathrm{RO}(G)$ determined by this virtual representation.

As regards equivariant orientability, there are several notions in the literature. Stong **[S3]** considered equivariant oriented bordism of oriented manifolds in which the group action preserves the orientation. This idea, which goes back to work of Conner and Floyd **[CF]** who studied oriented manifolds with actions by maps of odd period, appears in much of the subsequent work on oriented $G$-bordism. (See for example **[R1]**, **[R2]** and **[W4]**.) One also has the folklore requirement that the fixed-sets, together with their normal data, be "coherently" oriented, a weak version of this having appeared in work of Sebastiani and Rothenberg-Sondow **[S1]**, **[RS]**, in order to define equivariant connected sums. (See also **[B2; §VI.8]**.) The point here is that one wishes to avoid having to restrict to orientation-preserving actions.