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ON THE COHEN-MACAULAY PROPERTY IN COMMUTATIVE ALGEBRA AND SIMPLICIAL TOPOLOGY

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A ring R is called a "ring of sections" provided R is the section ring of a sheaf $(\mathscr{A}, X)$ of commutative rings defined over a base space X which is a finite partially ordered set given the order topology. Regard X as a finite abstract complex, where a chain in X corresponds to a simplex. In specific instances of $(\mathscr{A}, X)$, certain algebraic invariants of R are equivalent to certain topological invariants of X.

Introduction. The work of Reisner **[16]** shows a connection between the Cohen-Macaulay (CM) property in commutative algebra with a certain homological property of finite simplicial complexes. The purpose of this paper is to demonstrate a stronger connection. The main object of study in Reisner's Thesis is the face ring of a complex Σ with coefficients in a field F. In this paper the ring, hereby called the Stanley-Reisner ring and written $\mathrm{SR}(F, \Sigma)$, is also the main object of study.

The intent is to investigate the depth of factor rings of $\mathrm{SR}(F, \Sigma)$. The procedure is to regard $\mathrm{SR}(F, \Sigma)$ as the ring of sections of a sheaf of polynomial rings over a base space $X = X(\Sigma)$ where X is the partially ordered set of all simplices of Σ with order being reverse-inclusion. The method is to make statements about the depth of factor rings in the general section ring setting and then to particularize to the ring $\mathrm{SR}(F, \Sigma)$.

The homological property referred to in Reisner's Theorem **[16]** later proven to be a topological property **[13]** can be defined as follows. Let F be a field and Δ be a finite simplicial complex, or complex. Call Δ an *F-bouquet of spheres* if $\tilde{H}^i(\Delta, F) = 0$ for each $i < \dim \Delta$, the dimension of Δ, where $\tilde{H}^i(\Delta, F)$ denotes reduced singular cohomology with coefficients in F. A complex Σ is defined to be $\mathrm{CM}(F)$ provided the link subcomplex link (σ, Σ) is an *F-bouquet of spheres* for each $\sigma \in \Sigma$ (including $\phi \in \Sigma$).

Fix a field F. This paper shows $\mathrm{CM}(F)$ complexes are ubiquitous in the following sense. Let Σ be a complex with vertex set