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PSEUDOCONVEX CLASSES OF FUNCTIONS. II. AFFINE PSEUDOCONVEX CLASSES ON R^N

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A complete description of invariant pseudoconvex classes of functions on R^N which are closed with respect to addition of affine functions is given. Each such class is shown to be equal to its own bidual, and approximation results, including piecewise-smooth approximation and a counterexample to smooth approximation, are obtained. The results of the paper have applications to multivariate interpolation of normed spaces and to approximation of analytic multifunctions, which are given elsewhere.

Introduction. In this paper, which is a sequel to **[9]**, we continue to explore pseudoconvex classes of functions, a notion developed to provide conceptual framework and technical background for the study of multivariate interpolation methods for families of normed and quasi-normed spaces, which was undertaken in **[10]**.

Here, we restrict our attention to those pseudoconvex classes on R^N which are preserved by addition of linear functions and by translations. They will be called, shortly, *affine pseudoconvex classes*; axioms (0.1)–(0.9), listed below, comprise their precise definition.

Since the most important examples of pseudoconvex classes are, in fact, affine, and in view of the clarify of the methods required to analyse the Euclidean case, it seems worthwhile to obtain detailed description of the structure of affine pseudoconvex classes on R^N. This is the purpose of this paper.

In §2 the operation of supremum-convolution from [7] is used to approximate functions of a translation-invariant pseudoconvex class by functions of the same class which have almost everywhere second-order derivatives in the Peano sense.

This makes it possible to assign to every affine pseudoconvex class a nonempty set consisting of those $N \times N$ symmetric matrices which correspond to the Hessian forms of functions of the given class. In §3 it is proved that the set so obtained is closed and preserved by addition of positive-definite matrices. It is shown that such sets of matrices are in one-to-one correspondence with affine pseudoconvex classes of functions on R^N (cf. Theorem 3.11).