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THE POISSON FLOW ASSOCIATED WITH A MEASURE

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This paper is devoted to the study of harmonic functions on groups. The approach is via a detailed study of the Poisson flow associated with a Borel probability measure μ on a locally compact group T. Again the basic idea is that though many results associated with the study of harmonic functions on groups are couched in probabilistic terms and proved using methods of probability theory, they really belong in the domain of topological dynamics. The major results include a proof that a solvable connected Lie group admits only constants as harmonic functions for a spread out measure μ with $\mu(A) = \mu(A^{-1})$ for all Borel sets A, and a new non-geometric proof of a fundamental result of Furstenberg's on semi-simple Lie groups.

0. Introduction. The technical aspects of the paper depend on the methods and results developed in **[E]** and **[D]**. For the sake of completeness these are summarized in §1.

In §2 another approach to the Poisson flow is given. Let $\mathscr{R}$ be the algebra of right uniformly continuous functions on T, $|\mathscr{R}|$ its Gelfand space and $\mathscr{L}(\mu)$ the set of idempotent measures ν on $|\mathscr{R}|$ stationary with respect to μ and having the same harmonic functions. If the support of μ is all of T, the support S of ν is a subflow of $|\mathscr{R}|$. The main result of this section is that in this case there exists $\nu \in \mathscr{L}(\mu)$ such that the restriction $R\colon \mathscr{R} \to C(S)$ maps the set $\mathscr{H}_\mu$ of μ-harmonic functions isometrically onto a uniformly closed T-invariant subalgebra $\mathscr{K}_\mu$ of $C(S)$. The Poisson flow (B, T) is just the Gelfand space of $\mathscr{H}_\mu$. This has several implications, among them that $\omega p \in B$ $(p \in S)$ where ω is the measure on B induced by ν. Moreover the algebra of the flow $(\overline{\nu T}, T)$ is isomorphic to the subalgebra of $\mathscr{R}$ generated by $\mathscr{H}_\mu$ and (B, T) is the subflow of $(\overline{\nu T}, T)$ given by $B = \{\nu p | p \in S\}$. In this paper the results of this rather technical section are used only in §5.

Another aim of this paper is to obtain conditions under which a subgroup K of T will act transitively on B. The particular case $K = \{e\}$ says that the constants are the only μ-harmonic functions.

Sections 3 and 4 are devoted to this issue. In the former, conditions are studied which suffice to guarantee that a particular element of T