CHARACTERIZATIONS OF CONDITIONAL EXPECTATION-TYPE OPERATORS

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A complete description of positive projections in ideals of measurable functions is given in terms of conditional expectation-type operators.

1. Introduction. As is well-known, conditional expectation operators on various function spaces exhibit a number of remarkable properties related either to the underlying order structure of the given function space, or to the metric structure when the function space is equipped with a norm. Such operators are necessarily positive projections which are averaging in a precise sense to be described below and in certain normed function spaces are contractive for the given norm. Conditional expectation operators on L^p-spaces have been characterized in terms of their averaging property by Moy [M] and Rota [Ro] and as contractive projections by Douglas [D1] and Ando [A].

More recently, positive projections of a certain class of Banach function spaces have been characterized in terms of conditional expectation by Kulakova [Ku]. It is our intention in this paper to show that the characterization of positive projections given by Kulakova remains valid for arbitrary ideals of measurable functions. The method of [Ku] is based on the approach of Douglas for the case of L^1-spaces and makes essential use of the underlying metric structure via an appeal to a well-known interpolation theorem for rearrangement invariant KB-spaces. The approach to the present paper is on the other hand purely algebraic and uses the underlying order structure via a suitable adaptation of the ideas of Moy on averaging operators. The basic link which in essence reduces the more general problem of describing positive projections to the study of averaging operators is provided by a result of Kelley [K] which implies that each positive projection on an L^∞-space with range a vector sublattice containing the constants is necessarily averaging.

After gathering some preliminary notions in §2, we consider in §3 the relation between conditional expectations and averaging operators