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THE n-DIMENSIONAL ANALOGUE OF THE CATENARY: EXISTENCE AND NON-EXISTENCE

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We study "heavy" n-dimensional surfaces suspended from some prescribed $(n-1)$-dimensional boundary data. This leads to a mean curvature type equation with a non-monotone right hand side. We show that the equation has no solution if the boundary data are too small, and, using a fixed point argument, that the problem always has a smooth solution for sufficiently large boundary data.

Consider a material surface M of constant mass density which is suspended from an $(n-1)$-dimensional surface Γ in $\mathbb{R}^n \times \mathbb{R}^+$, $\mathbb{R}^+ = \{t > 0\}$, and hangs under its own weight. If M is given as graph of a regular function $u\colon \Omega \to \mathbb{R}^+$ on a domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, then u provides an equilibrium for the potential energy $\mathscr{E}$ under gravitational forces,

$$\mathscr{E}(u) = \int_\Omega u\sqrt{1+|Du|^2}.$$

Thus u solves the Dirichlet problem

$$\begin{aligned} &\operatorname{div}\left\{\frac{u\cdot Du}{\sqrt{1+|Du|^2}}\right\} = \sqrt{1+|Du|^2} \qquad &&\text{in } \Omega, \\ &u = \varphi &&\text{on } \partial\Omega \end{aligned} \tag{1}$$

The corresponding variational problem

$$\int_\Omega u\sqrt{1+|Du|^2} + \frac{1}{2}\int_{\partial\Omega} |u^2 - \varphi^2|\, d\mathscr{H}_{n-1} \to \min \tag{2}$$

in the class

$$BV_2^+(\Omega) := \{u \in L_2(\Omega)\colon u \geq 0,\ u^2 \in BV(\Omega)\}$$

has been solved by Bemelmans and Dierkes in **[BD]**. It was shown in [**BD**, Theorem 7] that the coincidence set $\{u = 0\}$ of a minimizer u is non-empty provided that

$$|\varphi|_{\infty,\partial\Omega} < \frac{|\Omega|}{\mathscr{H}_{n-1}(\partial\Omega)}, \tag{3}$$