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GENERIC PROPERTIES OF THE ADJUNCTION MAPPING FOR SINGULAR SURFACES AND APPLICATIONS

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In the last years many new results on the classical problem of classifying smooth surfaces in the projective space in terms of their extrinsic projective and intrinsic geometric invariants have been made by using the adjunction mapping. In this paper we extend the existence theorem for the adjunction mapping to the case of singular surfaces. Although the mapping is only meromorphic we obtain many inequalities known previously only in the smooth case. As an illustration of the results we given a very complete answer in the singular case, parallel to the smooth result, to the question of when a singular surface can "have a hyperelliptic hyperplane section".

Introduction. In modern times Sommese **[S1]** introduced the adjunction mapping to attack this problem. This line of attack led to a complete solution of this question in **[S1]**, **[V]**, **[S2]**, **[E]**, **[Se]**, **[S-V]**. The methods introduced have had wide applicability to the solution of classification questions for projective manifolds; see **[S5]**, **[S-V]** for applications and references.

We are interested in how much of the smooth technique survives when no smoothness assumptions are made. As a test we pose and give a surprisingly complete answer to the following problem.

Problem. Let Σ be an irreducible complex surface embedded in some complex projective space $\mathbb{P}^r$ and $\eta\colon S \to \Sigma$ be the normalization of Σ with L the pullback of S under η of $\mathcal{O}_{\mathbb{P}^r}(1)$. If there is a smooth hyperelliptic curve $C \in |L|$, the linear system associated to L, then describe (S, L).

The main tool we use is the meromorphic map associated to $\Gamma(K_S \otimes L)$, which is called the *adjunction map*. In **[A-S1]**, the first and last author showed that if $\pi\colon S' \to S$ is the minimal desingularization of S, then $K_{S'} \otimes L'$, $L' = \pi^* L$, is nef and big except when (S', L') and (S, L) are of very restricted type. In the case when $K_{S'} \otimes L'$ is nef, $h^0(K_{S'} \otimes L') \neq 0$ (see (0.6), (0.7)). These results allow us to