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PIECEWISE SMOOTH APPROXIMATIONS TO q-PLURISUBHARMONIC FUNCTIONS

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It is shown that q-plurisubharmonic functions can be approximated by piecewise smooth q-plurisubharmonic functions, and that analytic multifunctions are intersections of analytic multifunctions whose graphs are unions of complex analytic manifolds of the appropriate dimensions.

1. Introduction. This research is an outgrowth of an attempt to answer a question raised by Ted Gamelin in lectures delivered at the University of Washington in the spring of 1986: Can an analytic multifunction on an open set W in $\mathbb{C}$, whose values are subsets of $\mathbb{C}^n$, be approximated from above by analytic multifunctions whose graphs are unions of analytic disks? The question is intimately related to approximating $(n-1)$-plurisubharmonic functions on $W \times \mathbb{C}^n$ by functions of the same type which are smooth enough to allow construction of the disks.

A smooth C^2 function on an open set in $\mathbb{C}^n$ is said to be q-plurisubharmonic $(0 \leq q \leq n-1)$ if its complex Hessian has at least $(n-q)$ non-negative eigenvalues everywhere. The concept was introduced by Andreotti and Grauert **[AG]**, who call these functions $(q+1)$-plurisubharmonic. A broader definition that extends the notion to upper semicontinuous functions was given by Hunt and Murray **[HM]**, who also seem to be responsible for changing the index q. We will here follow the Hunt and Murray convention to minimize confusion.

The class of q-plurisubharmonic functions is not additive for $q > 0$ and thus standard smoothing techniques available for plurisubharmonic functions do not carry over, a fact which hampered early work on the subject. A breakthrough was achieved by Slodkowski **[S1]** who was able to show that continuous q-plurisubharmonic functions are uniform limits of functions whose second order derivatives exist almost everywhere. We will show that the approximation can be achieved by functions which are locally the maximum of a finite number of smooth (strictly) q-plurisubharmonic functions.