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DIAGONAL STATES ON $\mathcal{O}_2$

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Introduction. In this paper we investigate certain states on the Cuntz algebra $\mathcal{O}_2$, and the von Neumann algebras obtained from their GNS representations. The problem we begin with is that of finding different types of factor states on $\mathcal{O}_2$ which extend the trace on Choi's algebra, viewed as a subalgebra of $\mathcal{O}_2$. The existence in general of such factor state extensions was established independently by Longo and Popa (see [1]).

The construction of specific examples, and classifying them as to type, has been done by several mathematicians. These examples arise by composing the expectation of $\mathcal{O}_2$ onto the CAR algebra with a factor state on the CAR algebra. Work of Evans and Lance showed that by starting with the trace on the CAR algebra, a type $\mathrm{III}_{1/2}$ factor state extension is obtained (see [1]). In [9], pure state extensions are constructed by a combinatorial argument. In [2], it is shown that uncountably many inequivalent pure state extensions are so obtained, and that they arise from certain pure states on the CAR algebra. Moreover, it is shown that if the Powers III_λ states on the CAR algebra are extended to $\mathcal{O}_2$, they result in factor states of type III_λ if $\lambda^{n+1}+\lambda^n = 1$, some n in $\mathbb{Z}_+$, or III_1, if $\log\lambda$ and $\log(\lambda+1)$ are algebraically independent. In [11], a different collection of product states on the CAR algebra is shown to give rise to factor state extensions of type III_λ for all $0 \leq \lambda \leq 1$.

The techniques in [2] and [11] rely on the quasi-invariance, under the shift automorphism of [6], of an appropriate state or weight on the stabilized CAR algebra. In §1 of the present paper it is shown that arbitrary (infinite) Krieger factors can be obtained from factor state extensions on $\mathcal{O}_2$. The technique is the opposite of the above: namely, we use weights all of whose translates by powers of the shift are disjoint.