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REPRESENTATIONS OF BRAID GROUPS AND THE QUANTUM YANG-BAXTER EQUATION

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We are going to study the construction of new representations of braid groups and solutions of quantum Yang-Baxter (=QYBE) from existing ones via cabling. This can be applied for the construction of new link invariants from a given one for a wide class of invariants. For the example of the 2-variable generalization of the Jones polynomial, this yields for each Young diagram a 1-parameter family of representations of the braid groups and a 2-variable link invariant. Using the braid representations from the QYBE, one obtains a 1-variable link invariant for each irreducible representation of a classical Lie algebra.

It is well known that one obtains an injective homomorphism from B_∞ (which can be thought of as the inductive limit of the finite braid groups) into itself by cabling, i.e. by replacing each string of a braid $\beta \in B_\infty$ by f parallel strings (see figures in §1). Composing this homomorphism with a given representation ρ of B_∞ yields a new representation of $\rho^{(f)}$ of B_∞. Similarly as with higher tensor product representations, one can decompose the cabled representation of B_∞. If $\rho(\mathbf{C}B_f)$ is finite dimensional, the subrepresentations are given in terms of minimal idempotents of $\rho(\mathbf{C}B_f)$. More representations can be constructed by applying additional twists on the cabled braids. We will show that while the full representations may be different special subrepresentations are the same for all these twistings.

In the approach by Jones, link invariants were constructed by finding special traces on $\mathbf{C}B_\infty$, the so-called Markov traces. On the other hand, each link invariant with values in a field k and a minor additional condition can be used to define a Markov trace on kB_∞. Moreover, there is a well-known construction (known to operator algebraists as GNS construction) by which one obtains a representation of B_∞ from a Markov trace. We will apply this correspondence between link invariants, Markov traces and representations of braid groups to construct new link invariants from existing ones via cabling and decomposition of the corresponding braid representations. Essentially the same observation has been made by J. Murakami in his approach of computing cabled line invariants (see [M]). In this section,