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THE COUPLED YANG-MILLS-DIRAC EQUATIONS FOR DIFFERENTIAL FORMS

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A version of the coupled Yang-Mills-Dirac equations for differential forms is presented. In this version the equations are defined and conformal in any *odd* dimension; they share many of the analytic properties of the Yang-Mills-Higgs equations in these dimensions. A point singularity problem is formulated and solved for the Yang-Mills-Dirac equations in dimension 3. In this dimension the solutions can be associated with a definite energy functional resembling the magnetic-monopole energy.

1. The Yang-Mills-Dirac equations are a coupled system of nonlinear partial differential equations in which the unknowns are sections of twisted vector bundles. When the base space is $\mathbb{R}^4$ these equations can be used to describe the interaction of an external force field (the Yang-Mills field F_A) with the field φ induced by a fermion (the Dirac field).

In dimensions $n \neq 4$ the equations have mathematical interest as an example of a system which is elliptic modulo the action of a symmetry group. That is, let (A, φ) be a solution of the Yang-Mills-Dirac equations and let G be a fixed transformation group. Then the pair $(g(A), g(\varphi))$, $g \in G$, is also a solution and we identify (A, φ) with $(g(A), g(\varphi))$. There is a $g_0 \in G$ such that $(g_0(A), g_0(\varphi))$ is a solution of an *elliptic* system of partial differential equations, although the system satisfied by (A, φ) may not be elliptic.

In such a model the removability of singularities is a particularly interesting problem for two reasons. First, the Yang-Mills-Dirac equations have an interpretation as the Euler-Lagrange equations of an energy functional, so ordinarily one would consider weak solutions and show the existence of classical solutions by a partial regularity argument. Unfortunately, the concept of a weak solution is ambiguous in this case, since (A, φ) and $(g(A), g(\varphi))$ need not lie in the same function spaces. Thus one is motivated to try to characterize the singular set by asking what kind of singularities a classical solution can have. Second, the transformation g_0 may act discontinuously on the fiber, changing the topology of the vector bundle and thus altering the