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POINCARÉ COBORDISM EXACT SEQUENCES AND CHARACTERISATION

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Exact sequences connecting oriented and unoriented cobordism groups of Poincaré duality spaces, analogous to Rohlin's and Wall's exact sequences in differential and piecewise linear categories, are established and characterisation of elements of Poincaré cobordism groups (both oriented and unoriented) in terms of spherical characteristic numbers and index are given.

Introduction (Notations and Main results). By a Poincaré duality space we will mean a finite complex satisfying Poincaré duality with local coefficients in the sense of [27]. Corresponding to each P. D. space X^n of dimension n, there is a k-spherical fibration ν_X^k ($k \gg n$, i.e., ν_X is a stable fibration) unique up to fibre homotopy equivalence, and a commutative diagram:

$$\begin{array}{ccc} E(\nu_X) & \longrightarrow & E(\gamma_{BG}) \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & BG, \end{array}$$

where $\gamma_{BG} : E(\gamma_{BG}) \to BG$ is the stable universal unoriented spherical fibration and f is the classifying map of ν_X. Let $\gamma_{BSG} : E(\gamma_{BSG}) \to BSG$ be the stable universal oriented spherical fibration (BSG is obtained from BG by killing the first Stiefel-Whitney class). If ν_X can be classified by γ_{BSG} then we call X an oriented P.D. space. We now define various Poincaré cobordism groups. Suppose $\eta^k : E(\eta) \to B$ is a k-spherical fibration. We define $T(S^{n+k}, T(\eta^k))$ to be the set of cobordism classes of triples (X^n, f, b), where X^n is a P. D. space and f, b are base and total space maps, respectively, in the following diagram

$$\begin{array}{ccc} E(\nu_X) & \xrightarrow{b} & E(\eta) \\ \downarrow & & \downarrow \\ X & \xrightarrow{f} & B. \end{array}$$