

MULTIPLIERS OF H^p AND BMOA

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We characterize the multipliers from H^1 to X , where X is BMOA, VMOA, \mathcal{B} or \mathcal{B}_0 and from H^p to H^q ($p < \min(q, 1)$). Also we give short proofs of some results of Hardy and Littlewood and Fleet.

I. Introduction. For $0 < p \leq \infty$, by H^p we denote the space of functions $f(z)$ analytic in the unit disk U , for which

$$M_p^p(r, f) = \frac{1}{2\pi} \int_0^{2\pi} |f(re^{i\theta})|^p d\theta$$

or

$$M_\infty(r, f) = \max_{0 \leq \theta \leq 2\pi} |f(re^{i\theta})|$$

remains bounded as $r \rightarrow 1$. Duren's book [4] and Garnett's book [11] will be frequently cited as a reference to H^p theory and related subjects.

Let A and B be two vector spaces of sequences. A sequence $\lambda = \{\lambda_n\}$ is said to be a multiplier from A to B if $\{\lambda_n \alpha_n\} \in B$ whenever $\{\alpha_n\} \in A$. The set of all multipliers from A to B will be denoted by (A, B) . We regard spaces of analytic functions in the disk as sequence spaces by identifying a function with its sequence of Taylor coefficients.

Hardy and Littlewood [14] have proved the following theorem: If $1 \leq p \leq 2 \leq q$ and $p^{-1} - q^{-1} = 1 - \sigma^{-1}$ and if

$$(1.1) \quad M_\sigma(r, g') \leq c(1-r)^{-1}, \quad 0 < r < 1,$$

then $g \in (H^p, H^q)$ (c will be used for a general constant, not necessarily the same at each occurrence). Stein and Zygmund [21] (see also Sledd [20]) have observed that the condition (1.1) is also necessary in the case $p = 1$, $q \geq 2$. Hence the following theorem holds.

THEOREM HL. *Let $2 \leq q < \infty$. Then $g \in (H^1, H^q)$ if and only if*

$$(1.2) \quad M_q(r, g') \leq c/(1-r), \quad 0 < r < 1.$$