A FORMULA FOR SEGRE CLASSES OF SINGULAR PROJECTIVE VARIETIES

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The Segre class of a singular projective variety X is that of the normal cone of the diagonal in the product $X \times X$. This class was introduced by K. W. Johnson and W. Fulton to study immersions and embeddings. In our previous work we related the Segre classes and the Chern-Mather classes for hypersurfaces with codimension one singularities and $X^n \subset \mathbb{P}^{2n}$ with isolated singularities. In this paper we generalize these results to the case of $X^n \subset \mathbb{P}^N$ with singularities of codimension N - n $(N \leq 2n)$.

The notion of Segre classes (of cones) has become of increasing importance as a key ingredient for constructing or analyzing various invariants, e.g., in intersection theory and group representation theory, etc.

The Segre class treated in this note is the Segre class of a singular projective variety, which was introduced by K. W. Johnson (and W. Fulton) to study immersions and embeddings of singular projective varieties [4]. This is the Segre class of the normal cone $C_{\Delta}(X \times X)$ of the diagonal Δ in the product $X \times X$. We call this class Johnson's Segre class, denoted by $S_*(X)$.

Another well-studied characteristic class of a singular variety is MacPherson's Chern class, the existence of which was conjectured by Deligne and Grothendieck. R. MacPherson [7] constructed this Chern class, using Chern-Mather classes and introducing the notion of local Euler obstruction. A. Dubson [2] gave a more concrete description for MacPherson's Chern class $C_*(X)$: Let \mathscr{S}_X be a (in fact, any) Whitney stratification of X with the smooth part of X as the top-dimensional stratum and let $C_*^M(X)$ denote the Chern-Mather class of X. Then

$$C_*(X) = C^M_*(X) + \sum_{\substack{S \in \mathscr{S}_X \\ \dim S < \dim X}} m_S \cdot C^M_*(\overline{S}),$$

where m_S is a certain integer attached to each stratum S.

Motivated by Dubson's formula relating MacPherson's Chern class and the Chern-Mather class, the author [9] introduced the Segre-