

## SYMPLECTIC-WHITTAKER MODELS FOR $GL_n$

MICHAEL J. HEUMOS AND STEPHEN RALLIS

We consider the Klyachko models of admissible irreducible representations of the group  $GL_n(F)$  where  $F$  is a non-Archimedean local field of characteristic 0. These are models which generalize the usual Whittaker model by allowing the inducing subgroup a symplectic component. We prove the uniqueness of the symplectic models and the disjointness for unitary representations of the different models. Moreover, for  $n \leq 4$  we prove that all unitary irreducible representations admit a Klyachko model.

**Introduction.** Let  $F$  be a non-Archimedean local field of characteristic zero. This paper studies the realization of irreducible, admissible representation of  $GL_n(F)$  in certain induced representations generalizing the Whittaker model. In contrast to generalizing by allowing degenerate Whittaker characters or smaller unipotent groups arising from some degenerate data (cf. [Mo-Wa]), we generalize the inducing subgroup by allowing a symplectic component.

Our investigation is motivated by results of A. A. Klyachko [Kl], who exhibited a model, in the sense of I. M. Gel'fand, for  $GL_n$  over a finite field. He found a set of representations (which we will refer to as models) which are disjoint, multiplicity free and exhaust the set of irreducible representations. The representations he considers form a family  $\mathcal{M}_{n,k}$ ,  $0 \leq k \leq [\frac{n}{2}]$ . One extreme  $\mathcal{M}_{n,0}$ , is the Whittaker model, a representation induced off a character on the subgroup of unipotent, upper triangular matrices. When  $n$  is even, the other extreme  $\mathcal{M}_{n,n/2}$  is induced off the trivial character of  $Sp_n$ , the symplectic group of  $2n \times 2n$  matrices. The other "mixed" models  $\mathcal{M}_{n,k}$ ,  $0 < k < \frac{n}{2}$ , are induced off characters of subgroups coming from smaller unipotent and symplectic groups. Since the Whittaker model for representations of  $p$ -adic  $GL_n$  is of considerable importance, e.g. in the study of automorphic forms, it is natural to investigate the role of the other models in the  $p$ -adic case.

The natural category to study in the local field setting is the category of admissible representations. The Whittaker model  $\mathcal{M}_{n,0}$  is the only model which has received attention. It was shown by I. M. Gel'fand and D. A. Kazhdan ([Ge-Ka,1]) that the Whittaker model is unique,