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REMOVABLE SINGULARITIES FOR SUBHARMONIC FUNCTIONS

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Let Ω be an open set in $\mathbb{R}^n$ $(n \geq 3)$ and S be a C^2 $(n-1)$-dimensional manifold in Ω. Let $\alpha \in (0, n-2)$ and E be a compact subset of S of zero α-dimensional Hausdorff measure. We show that, if s is subharmonic in $\Omega \backslash E$ and satisfies $s(X) \leq c[\operatorname{dist}(X, S)]^{\alpha+2-n}$ for $X \in \Omega \backslash S$, then s has a subharmonic extension to the whole of Ω. The sharpness of this and other similar results is also established.

1. Introduction and results. Let Ω denote an open set in Euclidean space $\mathbb{R}^n$ $(n \geq 3)$, and let E be a compact subset of Ω. This paper is concerned with results of the following type: if s is a subharmonic function in $\Omega \backslash E$, where E is "small" and s is "not too badly behaved" (near E), then s has a subharmonic extension to the whole of Ω. We say in this case that E is a *removable singularity* of s. There is an obvious analogue of this notion for harmonic functions.

It is a consequence of a classical result [7, Theorem 5.18] that, if E is polar and s is a subharmonic function on $\Omega \backslash E$ which is bounded above near E, then E is a removable singularity of s. The idea behind our results is that, by imposing constraints on the geometry and size of the set E, the boundedness requirement can be considerably relaxed. The size of E is measured in terms of its α-dimensional Hausdorff measure $m_\alpha(E)$. A discussion of Hausdorff measures in relation to subharmonic functions can be found in Hayman and Kennedy [7, §5.4].

Let O_n denote the origin of $\mathbb{R}^n$, let $|X|$ denote the Euclidean norm of a point $X \in \mathbb{R}^n$, and $B(X, r)$ be the open ball of centre X and radius r. Also, let Φ: $\Omega \to \mathbb{R}$ be a C^2 function with nonvanishing gradient throughout Ω. We put $S = \{Y \in \Omega \colon \Phi(Y) = 0\}$.

THEOREM 1. *Let $\alpha \in (0, n-2)$ and E be a compact subset of S such that $m_\alpha(E) = 0$. If s is subharmonic in $\Omega \backslash E$ and satisfies*

$$s(X) \leq c[\operatorname{dist}(X, S)]^{\alpha+2-n} \qquad (X \in \Omega \backslash S) \tag{1}$$

for some positive constant c, then E is a removable singularity of s.