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A NOTE ON THE STABILITY THEOREM OF J. L. BARBOSA AND M. DO CARMO FOR CLOSED SURFACES OF CONSTANT MEAN CURVATURE

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The theorem of Barbosa and do Carmo asserts that the only stable compact hypersurface of constant mean curvature in R^{n+1} is the round n-sphere. We present an elementary proof of this fact by considering the 2-parameter family $y = s(x + t\xi)$ where x is the constant mean curvature immersion and ξ is the unit normal vector field.

I. Introduction. Let M be a compact oriented n-manifold and $x\colon M \to R^{n+1}$ an immersion of M into R^{n+1}. For such an immersion we compute the n-area $A(x)$

$$A(x) = \int_M dS \tag{1}$$

where dS is the n-area element on M induced by the immersion x. We can also compute the "oriented" volume $V(x)$ enclosed by the immersed surface $x(M)$. It is given by the formula

$$V(x) = \frac{1}{n+1}\int_M (x\cdot\xi)\,dS \tag{2}$$

where ξ is the unit normal vector field determined by the orientation of M and the immersion x.

Let $x_t\colon (-\varepsilon, \varepsilon) \times M \to R^{n+1}$ be a one-parameter family of immersions of M into R^{n+1} with $x_0 = x$. A necessary and sufficient condition that the area functional $A(x_t)$ have a critical value at $t = 0$ for all variations x_t for which $V(x_t)$ is constant is that the immersed surface have constant mean curvature H. Furthermore, such an immersion is said to be stable if for all volume-preserving perturbations the second derivative of $A(x_t)$ at $t = 0$ is non-negative.

In a recent paper [**1**] J. L. Barbosa and M. do Carmo proved the following theorem.

THEOREM [**1**]. *Let M be a compact oriented n-manifold and let $x\colon M \to R^{n+1}$ be an immersion with non-zero constant mean curvature*