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AUTOMATIC CONTINUITY OF DERIVATIONS AND EPIMORPHISMS

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Two automatic continuity problems for derivations on commutative Banach algebras are discussed: (a) Every derivation on a commutative Banach algebra maps into the radical, and: (b) Derivations on semiprime Banach algebras are continuous. It is shown that (b) implies (a). Further, (b) is reduced to a smaller class of Banach algebras and proved in some special cases. Related results for epimorphisms are given.

1. Introduction. A derivation on an algebra A is a linear mapping $D: A \to A$ that satisfies

$$D(ab) = aDb + (Da)b \qquad (a, b \in A).$$

In this paper we are concerned with derivations on commutative Banach algebras.

In 1955 Singer and Wermer (**[10]**) proved the now classical theorem: Every *continuous* derivation on a commutative Banach algebra maps into the radical. In the same paper they conjectured: (a) Every derivation on a commutative Banach algebra maps into the radical. In this context, it is reasonable to look for classes of Banach algebras on which every derivation is continuous. It was shown by B. E. Johnson that every derivation on a commutative, semisimple Banach algebra is continuous (**[5]**). Among the commutative Banach algebras the semisimple ones are characterized as those having no topologically nilpotent element other than zero; all known examples of discontinuous derivations on commutative Banach algebras depend crucially on the existence of nontrivial nilpotent elements in the algebraic sense ([7]). Thus, we might possibly generalize Johnson's theorem as follows: (b) Every derivation on a semiprime Banach algebra, i.e. on a commutative Banach algebra without nontrivial nilpotent elements, is continuous.

One result of this paper will be that (b) implies (a). Recently, (a) has been proved by Marc P. Thomas (**[11]**), whereas it still seems to be unknown whether (b) is true or not.