

ON INVARIANT SUBSPACES OF SEVERAL VARIABLE BERGMAN SPACES

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By using a natural localization method, one describes the finite codimensional invariant subspaces of the Bergman n -tuple of operators associated to some bounded pseudoconvex domains in \mathbb{C}^n , with a sufficiently nice boundary.

0. Introduction. Some recent investigations have been concerned with the structure and classification of the invariant subspaces of the Bergman n -tuple of operators, cf. Agrawal-Salinas [2], Axler-Bourdon [4], Bercovici [5], Douglas [7], Douglas-Paulsen [8]. Due to the richness of this lattice of invariant subspaces, the additional assumption on finite codimension was naturally adopted by the above mentioned authors as a first step towards a better understanding of its properties.

The present note arose from the observation that, when the L^2 -bounded evaluation points of a pseudoconvex domain lie in the Fredholm resolvent set of the associated Bergman n -tuple, then the description of finite codimensional invariant subspaces is, at least conceptually, a fairly simple algebraic matter. This simplification requires only the basic properties of the sheaf model for systems of commuting operators introduced in [11].

The main result below is also available by some other recent methods. First is the quite similar technique of localizing Hilbert modules over function algebras, due to Douglas [7] and Douglas and Paulsen [8], and secondly is the study of the so-called canonical subspaces of some Hilbert spaces with reproducing kernels, developed by Agrawal and Salinas [2]. Both points of view will be discussed in §2 of this note.

In fact the Bergman space of a pseudoconvex domain is only an example within a class of abstract Banach $\mathcal{O}(\mathbb{C}^n)$ -modules, whose finite codimensional submodules turn out to have a similar structure. The precise formulation of this remark ends the note.

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1. Pseudoconvex domains. Let Ω be a bounded pseudoconvex domain in \mathbb{C}^n , $n \geq 1$, and let $L_a^2(\Omega)$ denote the corresponding Bergman