PACIFIC JOURNAL OF MATHEMATICS
Vol. 147, No 2, 1991

THE HOMOLOGY OF A FREE LOOP SPACE

STEPHEN HALPERIN AND MICHELINE VIGUÉ-POIRRIER

Denote by X^{S^1} the space of all continuous maps from the circle into a simply connected finite CW complex, X. THEOREM: Let $\Bbbk$ be a field and suppose that either $\operatorname{char}\Bbbk > \dim X$ or that X is $\Bbbk$-formal. Then the betti numbers $b_q = \dim H_q(X^{S^1}; \Bbbk)$ are uniformly bounded above if and only if the $\Bbbk$-algebra $H^*(X; \Bbbk)$ is generated by a single cohomology class. COROLLARY: If, in addition, X is a smooth closed manifold and $\Bbbk$ is as in the theorem, and if $H^*(X; \Bbbk)$ is not generated by a single class then X has infinitely many distinct closed geodesics in any Riemannian metric.

1. Introduction. In this paper (co)homology is always singular and $b_q(-; \Bbbk) = \dim H_q(-; \Bbbk)$ denotes the *qth betti number* with respect to a field $\Bbbk$. The *free loop space,* X^{S^1}, of a simply connected space, X, is the space of all continuous maps from the circle into X.

The study of the homology of X^{S^1} is motivated by the following result of Gromoll and Meyer:

THEOREM **[16]**. *Assume that X is a simply connected, closed smooth manifold, and that for some field $\Bbbk$ the betti numbers $b_q(X^{S^1}; \Bbbk)$ are unbounded. Then X has infinitely many distinct closed geodesics in any Riemannian metric.*

(The proof in **[16]** is for $\Bbbk = \mathbb{R}$, but the arguments work in general.)

The Gromoll-Meyer theorem raises the problem of finding simple criteria on a topological space X which imply that the $b_q(X^{S^1}; \Bbbk)$ are unbounded for some $\Bbbk$. This problem was solved for $\Bbbk = \mathbb{Q}$ by Sullivan and Vigué-Poirrier **[28]**. They considered simply connected spaces X such that $\dim H^*(X; \mathbb{Q})$ was finite, and they showed that then the $b_q(X^{S^1}; \mathbb{Q})$ were unbounded if and only if the cohomology algebra $H^*(X; \mathbb{Q})$ was not generated by a single class. And they drew the obvious corollary following from the Gromoll-Meyer theorem.