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RINGS OF DIFFERENTIAL OPERATORS ON ONE DIMENSIONAL ALGEBRAS

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Let k be an algebraically closed field of characteristic zero, and A a finitely generated k-algebra of Krull dimension at most one. In this paper we study the ring of differential operators $\mathscr{D}(A)$. For example we obtain necessary and sufficient conditions for $\mathscr{D}(A)$ to be a direct sum of Simple rings, or to be left or right Noetherian.

0.1. Let k be an algebraically closed field of characteristic zero and A a finitely generated (commutative) k-algebra. The primary purpose of this paper is to study the ring $\mathscr{D}(A)$ of differential operators on A when $\dim(A)$, the Krull dimension of A, is at most one. If A is also reduced or is a domain $\mathscr{D}(A)$ has been studied extensively in **[10]** and **[15]** and we prove analogues of the main results of these papers. For example

THEOREM A. *Let A be a finitely generated k-algebra with Krull dimension at most one. Then*

(a) *$\mathscr{D}(A)$ is right Noetherian and finitely generated as a k-algebra.*

(b) *$\mathscr{D}(A)$ is left Noetherian if and only if A has an artinian quotient ring.*

0.2. One of the main ideas in **[15]** is to compare $\mathscr{D}(A)$, for A a domain, to $\mathscr{D}(\widetilde{A})$ where $\widetilde{A}$ is the integral closure of A. In particular, [**15**, Theorem B] gives necessary and sufficient conditions for $\mathscr{D}(A)$ and $\mathscr{D}(\widetilde{A})$ to be Morita equivalent. We prove a similar result here. We denote the nilradical of A by $N(A)$, and say that A has *injective normalisation* if every maximal ideal of $A/N(A)$ is contained in a unique maximal ideal of its integral closure.

THEOREM B. *Let A be a finitely generated algebra with* $\dim(A) \leq 1$ *and let $\widetilde{A}$ be the integral closure of $A/N(A)$. Then the following are equivalent:*

(1) *$\mathscr{D}(A)$ is Morita equivalent to $\mathscr{D}(\widetilde{A})$.*