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POINT SPECTRUM ON A QUASI HOMOGENEOUS TREE

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Dedicated to Professor Sh. Murakami on his 60th birthday

By using the algebraicity of the Green kernel it is shown that a linear operator of nearest neighbour type on a quasi homogeneous tree i.e. a tree admitting of a group of automorphism with finite quotient has no point spectrum on the space of square summable functions, provided the tree has a regular property and that the operator is invariant under the group of automorphism.

0. Introduction. This result is an extension of spectrum theorem on an anisotropic random walk on a homogeneous tree (see **[Ao1]** and **[Fi]**). In case of one dimensional lattice relevant results have been obtained in full generality (see **[Mo1]** and **[Mo2]**). See **[Ko]** for a similar problem on a Riemannian manifold. The author is indebted to the referee for various improvements of statements in this note. Among other things, in Theorem 1 the author has originally restricted himself to the graph Γ without loops and multiple edges. The referee has suggested the more complete present form with its proof.

1. Basic properties of the Green kernel. Let T be a connected locally finite tree with the set of vertices $V(T)$ and the set of edges $E(T)$. Let A be a symmetric operator on $l^2(T)$, the space of square summable complex valued functions on $V(T)$:

$$Au(x) = \sum_{\langle x, x'\rangle} a_{x,x'}u(x') + a_{x,x}u(x) \tag{1.1}$$

for $u(x) \in l^2(T)$, with $a_{x,x}$ and $a_{x,x'} = a_{x',x} \in \mathbb{R}$. $\langle x, x'\rangle$ means that two vertices x, x' are adjacent to each other with respect to an edge $\overline{x, x'}$ binding x and x'.

We assume first that A is regular in the following sense:

$$a_{x,x'} \neq 0 \quad \text{for all } \langle x, x'\rangle. \tag{$\mathscr{C}1$}$$

Suppose further that a discrete group of automorphism G of T acts fix point-freely on T:

$$G \times V(T) \ni (g, x) \to g \cdot x \in V(T) \tag{1.2}$$