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EXISTENCE AND GEOMETRY OF A FREE BOUNDARY PROBLEM FOR THE HEAT EQUATION

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A periodic (in t) free boundary problem for the one-dimensional heat equation is examined. The existence and regularity of the (unique) solution is established and the geometry of the free boundary is shown to be no more complicated than the geometry of the fixed boundary.

0. Introduction. Free boundary problems (and moving boundary problems) arise in a large variety of contexts and have been studied for over one hundred years. There is an extensive literature on many aspects of free boundary problems including the existence, uniqueness, regularity, and stability of solutions and the qualitative properties of the free boundary ([**14**], [**18**]). Many applications, especially to continuum mechanics, have been considered since the work in the 1860's of Helmholtz and of Kirchhoff on fluid jets and of Neuman on the Stefan problem ([**9**], [**10**], [**12**], [**15**], [**17**], [**27**], [**30**]). In addition, the approximation of solutions and of free boundaries using numerical methods is well established (e.g. [**16**]).

Among parabolic problems, Stefan problems have generated a great deal of interest and an extensive literature (e.g. [**11**], [**30**]). We will examine a periodic free boundary problem for the one-dimensional heat equation which might be considered as a free boundary problem of Stefan type ([**32**]) in which the known (or "fixed") boundary varies periodically in time and the free boundary is determined by a prescribed flux condition (rather than a phase-change condition). In addition, this can be viewed as a model for certain processes involving chemical reactions. Alternatively, our problem might be viewed as a model problem in which techniques which have proven useful for certain elliptic free boundary problems (e.g. [**1**], [**2**], [**5**]) are applied to a particular parabolic problem.

We will use a trial-free-boundary approach based on an operator method to establish the existence of a solution to our free boundary problem. Trial-free-boundary methods have been used for over 70 years with success, as illustrated, for example, by the work of Cryer