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OPERATORS PRESERVING DISJOINTNESS ON REARRANGEMENT INVARIANT SPACES

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Let X and Y be two rearrangement invariant spaces on a measure space (Ω, Σ, μ) with a finite, nonatomic measure μ. We show that if there exists a non-zero order continuous disjointness preserving operator $T: X \to Y$, then $X \subseteq Y$. This result has many consequences. For example, if $T: L_p(\Omega, \Sigma, \mu) \to L_q(\Omega, \Sigma, \mu)$ $(0 < p < q \leq \infty)$ preserves disjointness, then $T \equiv 0$.

1. Notation and preliminary facts. Recall that a (linear) operator $T: X \to Y$ between vector lattices is said to be a *disjointness preserving operator* if $|x_1| \wedge |x_2| = 0$ in X implies $|Tx_1| \wedge |Tx_2| = 0$ in Y. All vector lattices are assumed to be Archimedean, and all operators on normed or linear metric spaces are assumed to be continuous.

Let (Ω, Σ, μ) be a measure space with a finite σ-additive non-atomic measure and $S(\Omega, \Sigma, \mu)$ be the space of all (equivalence classes of) measurable real valued functions. Throughout the work we will use the representation of the space S as the space $C_\infty(Q)$ of all continuous extended functions on the Stone space Q of S. (See **[10]** for details.) We retain the same notation μ for the corresponding measure on Q, which is defined on the σ-algebra Σ_Q consisting of all subsets of the form $(E \backslash N) \cup (N \backslash E)$, where E is a *clopen* (closed and open) subset of Q and N is a first category subset of Q. It is well known that $\mu(D) = 0$ if and only if D is a nowhere dense subset of Q. (Any extremally disconnected space Q with such a measure is sometimes called a hyperstonian space.) A subspace X of $S(\Omega, \Sigma, \mu)$ is called a rearrangement invariant (r.i.) ideal if

(i) X is an order ideal in S, and

(ii) If $x \in X$, $y \in S$, and x and y are equimeasurable, in symbols $x \sim y$, then $y \in X$.

If, in addition, X is equipped with a Banach norm $\|\cdot\|$ such that

(iii) $x_1, x_2 \in X$ and $|x_1| \leq |x_2| \Rightarrow \|x_1\| \leq \|x_2\|$, and

(iv) $x_1, x_2 \in X$ and $x_1 \sim x_2 \Rightarrow \|x_1\| = \|x_2\|$,

then X is called a r.i. Banach function space. We refer to [7] for the basic facts concerning r.i. ideals and Banach spaces. (Let us mention