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GENERALIZED CLIFFORD-LITTLEWOOD-ECKMANN GROUPS

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This paper investigates the structure of "generalized Clifford-Littlewood-Eckmann groups", which arise in a number of physical applications. They are a direct generalization of Clifford-Littlewood-Eckmann groups, which have many connections to quadratic forms and classical Clifford algebras. Here we show that any such group decomposes into a central product of factor groups of relatively small order, and that the number of isomorphism types of these factor groups is also small. The determination of the decomposition of these groups allows an easy calculation of many of the properties of the groups as well as of their associated generalized Clifford algebras. These applications will be carried out in subsequent papers.

Introduction. In **[LS]** we analyzed the structure of those 2-groups which can be presented as $\mathbf{G} = \langle \varepsilon, a_1, \ldots, a_r | \varepsilon^2 = 1, a_i^2 = \varepsilon^{k(i)}\ \forall i, a_i a_j = \varepsilon a_j a_i\ \forall i < j, \varepsilon a_i = a_i \varepsilon\ \forall i \rangle$. Any group of this type can be parametrized in terms of the values $s := |\{i: k(i) \equiv 1 \pmod 2\}|$ and $t := |\{i: k(i) \equiv 0 \pmod 2\}|$, and can then be designated by $\mathbf{G} = \mathbf{G}_{s,t}$.

Examples of such groups (or closely related algebraic structures) have appeared in the mathematics and physics literature from the 19th century to the present day. For example, the so-called Dirac group is in fact $\mathbf{G}_{0,4}$, and more generally the groups $\mathbf{G}_{0,2n}$ arise naturally in quantum field theory (see, e.g., **[We]** and **[Lo]**). On the other hand, the groups $\mathbf{G}_{r,0}$ are exactly those used by Eckmann **[E]** in his elegant group-theoretic proof of the theorem of Hurwitz-Radon on the composition of sums of squares (**[H1]**, **[H2]**, **[R]**). Littlewood **[Li]** considered the general groups $\mathbf{G}_{s,t}$ in studying sets of anticommuting matrices.

These groups are implicit in Clifford's work on "geometric algebras" **[Cl]**. In fact, the group $\mathbf{G}_{s,t}$ appears naturally as a subgroup of the group of units of the Clifford algebra $C^{s,t}$ of the quadratic form $s\langle -1\rangle \perp t\langle 1\rangle$. These groups exhibit a "mod 8 periodicity" depending on s and t which parallels the well-known periodicity of the Clifford algebras; moreover, we can derive the Clifford algebra periodicity from that of the groups. Also the decomposition of the Clifford