PACIFIC JOURNAL OF MATHEMATICS
Vol. 149, No. 1, 1991

AN INTRINSIC CHARACTERIZATION OF A CLASS OF MINIMAL SURFACES IN CONSTANT CURVATURE MANIFOLDS

GENE DOUGLAS JOHNSON

Let X be an N-manifold of constant sectional curvature. A class of minimal surfaces in X, called exceptional minimal surfaces, will be defined in terms of the structure of their normal bundles. It will be shown that these surfaces can be characterized intrinsically in a way that generalizes the Ricci condition for minimal surfaces in Euclidean 3-space. It will also be shown that these surfaces are rigid when N is even and belong to 1-parameter families of isometric surfaces when N is odd.

0. Introduction. Let $X^N(c)$ denote an N-dimensional manifold of constant sectional curvature c, and suppose that M is a minimal surface in $X^N(c)$ with Riemannian metric ds^2 and Gauss curvature K. The classical theorem of Ricci, as extended by Lawson [3], says that when $N = 3$ minimal surfaces of $X^3(c)$ are characterized by the conditions that $K \leq c$ and at points where $K < c$ the metric $d\hat{s}^2 = \sqrt{c-k}\, ds^2$ is flat. Moreover, for each minimal surface M in $X^3(c)$, there is a 1-parameter family of isometric minimal surfaces M_τ, $0 \leq \tau < 2\pi$, such that M is congruent to one of the members of this family.

This paper will describe a class of minimal surfaces in $X^N(c)$, called exceptional minimal surfaces, and a sequence of functions A_1^c, A_2^c, ... on each surface such that when $N = 2n+1$, these surfaces are characterized by the conditions that $A_r^c \geq 0$, $1 \leq r \leq n$, and at points where each $A_r^c > 0$, the metric $d\hat{s}^2 = (A_n^c)^{1/(n+1)}\, ds^2$ is flat. This reduces to the Ricci-Lawson condition when $n = 1$, in that $A_1^c = c - K$. The exceptional minimal surfaces in $X^{2n+1}(c)$ will be seen to belong to 1-parameter families of isometric surfaces, just as happens in $X^3(c)$.

In $X^{2n+2}(c)$, the exceptional minimal surfaces will be characterized by the conditions that $A_r^c \geq 0$, $1 \leq r \leq n$, and $A_{n+1}^c \equiv 0$. Additionally, in $X^{2n+2}(c)$ the exceptional minimal surfaces will be rigid. These results given here for the case where $N = 2n+2$ are actually implicit in [2], although they are stated there in terms of minimal immersions of the 2-sphere S^2 into $X^{2n+2}(c)$.