PACIFIC JOURNAL OF MATHEMATICS
Vol. 149, No. 1, 1991

A MATRIX VOLTERRA INTEGRODIFFERENTIAL EQUATION OCCURRING IN POLYMER RHEOLOGY

HANS ENGLER

In this paper we study a class of Volterra integrodifferential equations that arise in the description of elastic liquids, such as polymer melts and dilute or concentrated polymer solutions. The deformation of a small cube-shaped sample of such a liquid can be approximately described by a symmetric 3×3-matrix: If the material undergoes some prescribed deformation for times $t \leq 0$ and then is allowed to recover without constraints for $t > 0$ (stress-free recoil), and if inertial effects are ignored, these matrices obey an ordinary first order Volterra integrodifferential equation. Incompressibility of the material imposes the nonlinear constraint that the determinant of the matrices remain constant. In addition, there is a natural small parameter $\eta > 0$, proportional to Newtonian viscosity, which multiplies the derivative. In the case $\eta = 0$, which is also of physical interest, the problem reduces to an implicit Volterra integral equation.

1. Introduction. The problem under study thus can be classified as a singularly perturbed Volterra integrodifferential equation on a manifold. In this paper we present an existence and uniqueness theory, asymptotic results (as $t \to \infty$) for the cases $\eta > 0$ and $\eta = 0$, and a study of the behavior of the solutions as $\eta \downarrow 0$. Thus, we follow the program of the influential paper **[11]**, in which these questions were studied for the case of elongational flows.

In the remainder of this section, the physical background of the problem is described, the equations to be studied are derived, and some special classes of deformations are listed for later reference. Section 2 reviews known results for the important class of elongational deformations. In §3, we develop some useful facts concerning the differential geometry of the manifold on which the equation holds. In §4, we show that the problem under study has a natural gradient structure and give a basic local existence and uniqueness theorem for the case $\eta > 0$. Section 5 deals with global existence and with the existence of asymptotic limits of solutions as $t \to \infty$, still for $\eta > 0$. In §6, we develop a variational framework in which local and global existence and uniqueness and the existence of asymptotic limits can also be proved for the reduced implicit equation that results from setting