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THE PROPER FORCING AXIOM AND STATIONARY SET REFLECTION

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Our main result is that the proper forcing axiom (PFA) is equiconsistent with "PFA + there is a nonreflecting stationary subset of ω_2." More generally we show for any cardinals $n < m \leq \aleph_2$ that if $\mathrm{PFA}^+(n)$ is consistent with ZFC then so is "$\mathrm{PFA}^+(n)$ + there are m mutually nonreflecting stationary subsets of ω_2." As corollaries we can show that if $n < m \leq \aleph_1$ then $\mathrm{PFA}^+(n)$ (if consistent) does not imply $\mathrm{PFA}^+(m)$, and that PFA (if consistent) does not imply Martin's's maximum.

1. Introduction. Recently much attention has been given to various strengthenings of Martin's axiom for $\aleph_1$. Following **[FMS]** let us denote by $\mathrm{MA}(\Gamma)$, where Γ is a class of partial orders, the statement:

> If $P \in \Gamma$ and Δ is a family of at most $\aleph_1$ dense subsets of P, then there is a Δ-generic filter on P.

Thus letting Γ be the class of all partial orders having the c.c.c., $\mathrm{MA}(\Gamma)$ becomes Martin's axiom (for $\aleph_1$ dense sets). Taking Γ to be the class of proper partial orders, $\mathrm{MA}(\Gamma)$ becomes the proper forcing axiom (PFA). Taking Γ to be the class of all orders P so that forcing the P preserves the stationarity of subsets of ω_1, $\mathrm{MA}(\Gamma)$ becomes Martin's maximum (MM), discussed in **[FMS]**.

One may fortify these axioms by demanding that the filter obtained not only be generic, but also respect the stationarity of a collection of subsets (in the generic extension) of ω_1. That is, one may consider the axioms

> $\mathrm{MA}^+(\Gamma, \kappa)$: If $P \in \Gamma$, Δ is a family of at most $\aleph_1$ dense subsets of P, and $\{\mathbf{S}_\alpha : \alpha < \kappa\}$ is a family of terms, each forced by every condition in P to denote a stationary subset of ω_1, then there is a Δ-generic filter G on P so that for every $\alpha < \kappa$, $\mathbf{S}_\alpha(G)$ is stationary in ω_1.

(Here $\mathbf{S}_\alpha(G) = \{\beta < \omega_1 : \exists p \in G\ p \Vdash \text{“}\beta \in \mathbf{S}_\alpha\text{”}\}$, the interpretation of the term $\mathbf{S}_\alpha$ by the filter G.) If Γ is the class of proper