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ON THE ROMANOV KERNEL AND KURANISHI'S L^2-ESTIMATE FOR $\overline{\partial}_b$ OVER A BALL IN THE STRONGLY PSEUDO CONVEX BOUNDARY

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As is proved by Kerzman-Stein, over a compact strongly pseudo convex boundary in C^n, Szegö projection S is the operator defined by Henkin-Ramirez *modulo compact operators*. While, over *a special ball*, U_ε, in the strongly pseudo convex boundary, in order to obtain a local embedding theorem of CR-structures, Kuranishi constructed the Neumann type operator N_b for $\overline{\partial}_b$ and so we have *a local Szegö operator* by

$$S_{U_\varepsilon} = \mathrm{id} - \overline{\partial}_b^* N_b \overline{\partial}_b \quad \text{on } U_\varepsilon,$$

where $\overline{\partial}_b^*$ means the adjoint operator of $\overline{\partial}_b$. There might be a relation between S_{U_ε} and the Romanov kernel like the case of the Szegö operator and the Henkin-Ramirez kernel. We study this problem and show some estimates for the Romanov kernel.

0. Introduction. Let $(M, {}^\circ T'')$ be an abstract strongly pseudo convex CR-manifold. Then as is well known, if $\dim_R M = 2n-1 \geq 7$, $(M, {}^\circ T'')$ is locally embeddable in a complex euclidean space $C^n((Ak3), (K))$. In the proof of this local embedding theorem, it is shown that: over *a special ball* in the strongly pseudo convex boundary, an L^2-estimate for $\overline{\partial}_b$, which is stronger than the standard L^2-estimate, is established and so the L^2-solution operator for $\overline{\partial}_b$ is obtained. This operator plays an essential role in our local embedding theorem. Therefore it must be important to study this solution operator for $\overline{\partial}_b$ precisely.

In order to get a solution operator, there exists another method. By using an integral formula, a local solution operator for $\overline{\partial}_b$ is constructed explicitly by Henkin and Harvey-Polking. Obviously, these solution operators are different. And it seems quite interesting to study the relation between the L^2-solution for $\overline{\partial}_b$ and the explicit solution, obtained by using an integral formula. We recall the $\overline{\partial}$-case over a strongly pseudo convex domain in C^n. In this case, the explicit solution, constructed by Lieb and Range, is a certain kind of the essential part of the Kohn's L^2-solution. Therefore we could hope for a similar result in the $\overline{\partial}_b$ case over *a special ball* in the strongly pseudo