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COHOMOLOGY OF ACTIONS OF DISCRETE GROUPS ON FACTORS OF TYPE II_1

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We study 1-cohomology of discrete group actions on factors of type II_1. Characterizations of Kazhdan's property T and amenability for discrete groups in terms of cocycles and coboundaries are given, and we show that each of $\mathrm{SL}(n, \mathbf{Z})$, $n \geq 3$, and $\mathrm{Sp}(n, \mathbf{Z})$, $n \geq 2$, has a continuous family of mutually non-cocycle conjugate free actions on the AFD factor of type II_1 as an application. We also introduce and compute entropy for discrete amenable group action on factors of type II_1.

0. Introduction. In this paper, we study 1-cohomology of discrete group actions on factors of type II_1. We give characterizations of Kazhdan's property T and amenability for discrete groups in terms of 1-cocycles and coboundaries for actions on factors of type II_1. As an application, we also show that each of $\mathrm{SL}(n, \mathbf{Z})$, $n \geq 3$, and $\mathrm{Sp}(n, \mathbf{Z})$, $n \geq 2$, has a continuous family of mutually non-cocycle conjugate ergodic free actions on the approximately finite dimensional (AFD) factor of type II_1. These are typical groups with Kazhdan's property T. We introduce and compute entropy of discrete amenable group actions on the AFD factor of type II_1.

Complete classification of actions of discrete amenable groups on the AFD factor $\mathscr{R}$ of type II_1 up to cocycle conjugacy was given in Ocneanu [O]. In particular, he showed that any two free actions of a discrete amenable group on $\mathscr{R}$ are cocycle conjugate. Then Jones [J2] showed that this statement is no longer valid for any discrete non-amenable group. He constructed two free actions and used the ergodicity at infinity to distinguish the two. This shows that non-amenable discrete groups are quite different from amenable ones in the theory of group actions on factors. In order to understand cocycle conjugacy of nonamenable group actions, we start to study 1-cohomology of the actions and get several von Neumann algebra analogues of Schmidt's work [S] on ergodic actions on probability spaces. A major difference between the cohomology theory on probability spaces and one on von Neumann algebras is that we do not have the group structure on the space of cocycles in the latter case, which causes technical difficulty.