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CHERN CLASSES AND COHOMOLOGY FOR RANK 2 REFLEXIVE SHEAVES ON $\mathbf{P}^3$

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The paper shows that, if F is a nonsplit rank 2 reflexive sheaf on $\mathbf{P}^3$, then the knowledge of the numbers $d_n = h^2(F(n)) - h^1(F(n))$ gives an explicit algorithm to compute the Chern classes c_1, c_2, c_3 and the dimensions $h^0(F(n))$, for all n (in particular the first integer a such that the sheaf $F(a)$ has some nonzero section). If the sheaf is a vector bundle it is also proved that the knowledge of the numerical sequence $\{h^1(F(n))\}$ together with the first Chern class gives all the information as above. In some special cases, i.e. when $h^1(F(n)) \neq 0$ for at most three values of n, an algorithm is also produced to compute the first Chern class from the sequence $\{h^1(F(n))\}$. Vector bundles with natural cohomology are also discussed.

It must be remarked that, if one knows not only the dimensions $h^1(F(n))$, for all n, but also the whole structure of the Rao-module $\bigoplus H^1(F(n))$, then the first Chern class c_1 is uniquely determined (as it is shown in a paper by P. Rao).

n1. F is a rank 2 *nonsplit* reflexive sheaf on $\mathbf{P}^3 = \mathbf{P}$. Its Chern classes are c_1, c_2, c_3; if it is normalized, then $c_1 = 0$ or -1. Once and for all $h^i(F(n)) = \dim F(n))$, $i = 0, 1, 2, 3$.

Now we give a list of well-known properties useful throughout the paper.

1. If $c_1(F) = c_1$, the associated normalized reflexive sheaf is defined as $F^n = F(\varepsilon)$ where

$$\varepsilon = \begin{cases} \dfrac{c_1}{2} & \text{for } c_1 \text{ even,} \\ -\dfrac{c_1+1}{2} & \text{for } c_1 \text{ odd.} \end{cases}$$

2. With every reflexive sheaf F there are two associated numbers:

$$a = a(F) = \text{smallest integer } n \text{ such that } h^0(F(n)) \neq 0,$$
$$a_1 = a_1(F) = \text{smallest integer } n \geq a \text{ such that } h^0(F(n)) > h^0(O_P(n-a)).$$

Since F is not split, then every general nonzero section of $F(a)$ gives rise to a zero locus which is necessarily a curve in $\mathbf{P}$ (see **[H1]**, n.1 and **[H2]**, n.4); this is false for a split sheaf. The same is true for