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SURGERY WITH FINITE FUNDAMENTAL GROUP II: THE OOZING CONJECTURE

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We determine characteristic class formulae for surgery problems over any compact oriented and closed manifold with finite fundamental group. This essentially evaluates the boundary maps in the surgery exact sequences

$$\cdots \xrightarrow{\partial} L_{n+1}^h(\mathbf{Z}\pi) \longrightarrow \mathscr{HT}(M^n) \longrightarrow [M^n, G/TOP] \xrightarrow{\partial} L_n^h(\mathbf{Z}\pi).$$

(The final step in determining ∂ is carried out in a sequel which represents joint work with I. Hambleton, L. Taylor, and B. Williams.) These formulae involve the L-genus of M, pullbacks of the classes K_{4i} and k_{4i+2} in $H^*(G/TOP)$ and characteristic classes of the universal covering of M (coming from $H^*(B_{\pi_1(M)})$). It turns out that only classes in the first three of these groups, $* = 1, 2, 3$, are needed. This can be interpreted as saying that only codimension 1, 2, and 3 submanifolds are needed to determine the surgery obstruction. In this form our result was originally conjectured twenty years ago and has become known as the oozing conjecture.

In [5] I showed that there are only four types of surgery obstruction possible for surgery problems over closed manifolds with finite π_1 when the problem is of the form

$$M \times K^{4i+2} \longrightarrow M \times S^{4i+2}.$$

In order to do this I introduced an intermediate L group $L_*^h(\mathbf{Z}(\zeta_3)\pi)$ and maps

$$e_{\zeta_3}: \Omega_*(B_\pi) \longrightarrow L_*^h(\mathbf{Z}(\zeta_3)\pi)$$

which factor the product formula above. Then I showed that there were only four types of classes in the image of e_{ζ_3} which could possibly map non-trivially into the surgery groups.

In this paper I obtain characteristic class formulae for the maps e_{ζ_3}, at least as regards the specific classes described above. First recall the main results of [5]:

THEOREM A. *Let π be a finite 2-group, then there are universal kernels $K_i \subset L_i^h(\mathbf{Z}(\zeta_3)\pi)$ with $L_0^h(\mathbf{Z}(\zeta_3)\pi)/K_0 = \mathbf{Z}/2$, natural with*